Tutorial

Ranking Mechanisms in Games

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MARIE CURIE ALUMNI

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SYNERGY

Applications of Ranking Mechanisms

- **Hierarchy**
- Winner in a live event
- Matchmaking
- Handycapping
- Performance Thresholds
- In-game Decisions

Bundesliga standings

MATCHES NEWS STANDINGS STATS PLAYERS

	Club	MP	W	D	L	GF	GA	GD	Pts
1	Dortmund	34	23	6	5	67	22	45	75
2	Bayer	34	20	8	6	64	44	20	68
3	Bayern	34	19	8	7	81	40	41	65
4	Hannover 96	34	19	3	12	49	45	4	60
5	Mainz	34	18	4	12	52	39	13	58

Football (German Bundesliga 2011)

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StarCraft II (Grandmaster League 18 Season 2)

Applications of Ranking Mechanisms

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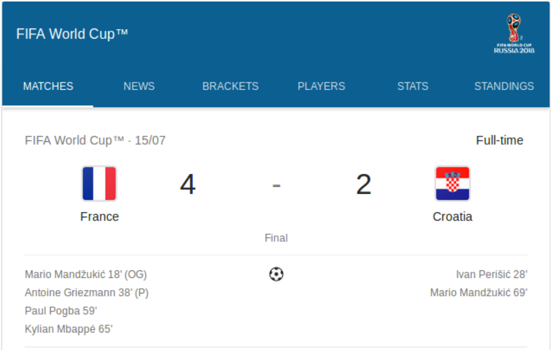

Football (FIFA World Cup 2018)

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AlphaGo vs. Lee Sedol (2016)

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Hearthstone Queue

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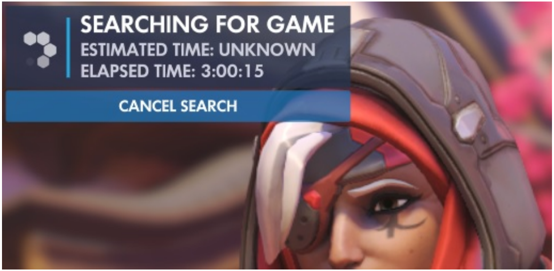

Overwatch Queue

Applications of Ranking Mechanisms

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Chess

Applications of Ranking Mechanisms

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Golf

Applications of Ranking Mechanisms

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National Collegiate Counter-Strike League

Applications of Ranking Mechanisms

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Smite Divisions

Applications of Ranking Mechanisms

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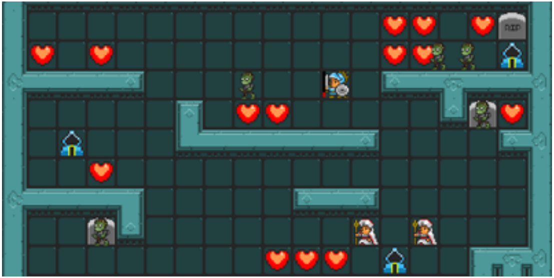
- Handycapping
- Performance Thresholds
- **In-game Decisions**



GVGAI

Ranking Mechanisms in CIG 18 Competitions

- Round Robin Tournament
  - Hearthstone AI
  - Fighting Game AI (Standard)
  - microRTS
  - StarCraft AI
- Average Score
  - Hanabi
  - Ms. Pac-Man vs. Ghost Team
  - Text-based adventure AI
  - Visual Doom AI (Deathmatch)
  - GVGAI
- Time to beat opponent
  - Fighting Game AI (Speedrun)
  - Visual Doom AI (Speedrun)
- Others
  - Short Video (Vote)
  - Hearthstone AI alt (Glicko2)
  - AI Birds: AI (Elim. tournament)
  - AI Birds: Level (Vote)

Why we're here!

- Various examples of ranking mechanisms in games
- But are they fair?

Social Choice Theory

- formalisation of characteristics
- recommendations for ranking mechanisms

Relations

- Relation R on a set X
  Subset of cartesian product $X \times X$:

$$R \subset X \times X$$

- Properties of relations
  - **reflexive**, if $\forall x \in X: \quad xRx$.
  - **symmetric**, if $\forall x, y \in X: \quad xRy \;\Rightarrow\; yRx$.
  - **anti-symmetric**, if $\forall x, y \in X: \quad xRy \;\wedge\; yRx \;\Rightarrow\; x = y$.
  - **transitive**, if $\forall x, y, z \in X: \quad xRy \;\wedge\; yRz \;\Rightarrow\; xRz$.

Examples for relations

Set of real number $\boldsymbol{R}$ and relation

"$<$" (less than)

- not reflexive ($x < x$ doesn't hold)
- not symmetric (from $x < y$ does not follow $y < x$)
- but anti-symmetric ($x < y$ and $y < x$ cannot hold both, hence implication is true)
- and transitive, (from $x < y$ and $y < z$ follows $x < z$)

"$\leqslant$" (less or equal)

- is reflexive ($x \leqslant x$ holds)
- not symmetric (in general $x \leqslant y$ does not imply $y \leqslant x$)
- but anti-symmetric ($x \leqslant y$ and $y \leqslant x$ implies $x = y$)
- and transitive (from $x < y$ and $y < z$ follows $x < z$)

Examples for relations

Set of real number $\boldsymbol{R}$ and relation

"$\neq$" (unequal)

- not reflexive ($x \neq x$ does not hold)
- but symmetric ($x \neq y \;\Rightarrow\; y \neq x$)
- not anti-symmetric ($x \neq y$ and $y \neq x$ do not imply $y = x$)
- and not transitive ($x \neq y$ and $y \neq z$ do not imply $x \neq z$; $x = z$ is still possible).

"$=$" (equal)

- is reflexive ($x = x$ holds)
- symmetrisch ($x = y \;\Rightarrow\; y = x$)
- anti-symmetric ($x = y$ and $y = x$ implies $x = y$)
- and transitive ($x = y$ and $y = z$ imply $x = z$)

Orders

- Relation R on set X is called **order** $:\Leftrightarrow$ R is
  - reflexive
  - anti-symmertric and
  - transitive
- Relation R on set X is called **linear** or **total order** $:\Leftrightarrow$ R is
  - an order
  - additionally:

$$\forall x, y \in X : \quad xRy \ \vee \ yRx$$

- Example
  - $(\boldsymbol{R}, <)$ is not an order, not reflexive
  - $(\boldsymbol{R}, \leqslant)$ is a total order

The social choice model

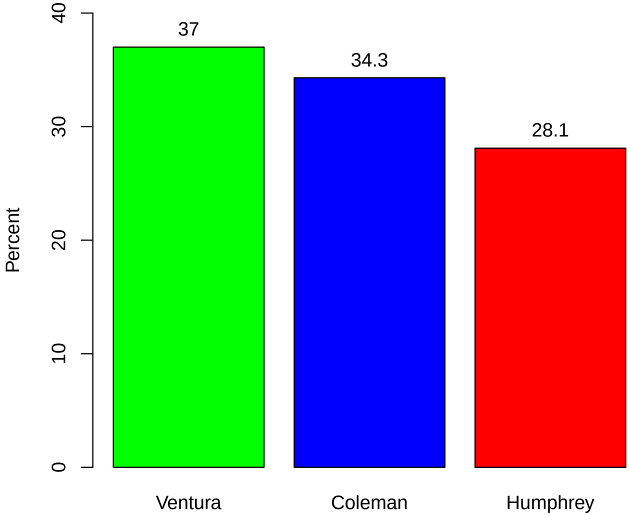
Social Choice Theory

- formalisation of characteristics
- recommendations for ranking mechanisms

How they correlate ...

- Finite set of n voters and finite set X of k choices or candidates
- In gaming competitions: n games and k players
- In racing competitions: n tracks and k drivers
- In algorithm comparision: n runs of k algorithms

1998 Minnesota governor election



Common Social Choice Example

Candidate	Votes
Jesse Ventura	37.0%
Norm Coleman	34.3%
Skip Humphrey	28.1%

Preference list			Perc. of voters
Coleman	Humphrey	Ventura	35%
Humphrey	Coleman	Ventura	28%
Ventura	Coleman	Humphrey	20%
Ventura	Humphrey	Coleman	17%

Ventura won, but

- 63% of voters liked him least!
- Coleman wins pairwise comparisons
  - 55% prefer Coleman to Humphrey
  - 63% prefer Coleman to Ventura

Easy example

- Imagine a racing competition featuring 7 tracks
- 3 drivers compete against each other: driver1, driver2, driver3

Preference list			Number of occurrences
driver1	driver2	driver3	3
driver2	driver1	driver3	2
driver3	driver2	driver1	2

- Who is the best driver?

And the winner is ...

Preference list			Number of occurrences
driver1	driver2	driver3	3
driver2	driver1	driver3	2
driver3	driver2	driver1	2

- **driver1 !**
  - Wins on most tracks
- **driver2 !**
  - Outperforms driver1 on 4 of 7 tracks

What?!? How ... ?!? 😱

The social choice model

- $L(X)$ set of all preference lists
  i.e. set of all possible strict linear orders of X (no ties allowed)
- $O(X)$ set of all preference lists
  i.e. set of all possible linear orders of X (ties allowed)
- Profile or election is element of cartesian product $L(X)^n$
  i.e. set of n preference lists, one from each voter (game, track)
- Ranking mechanism in games (social choice function or voting method) is function

  $$F : L(X)^n \rightarrow O(X).$$

  For given profile $R \in L(X)^n$, image $F(R)$ is called the ranking (social choice or societal ranking)

Examples of social choice functions, rankings

- Plurality (also called majority)
  - Candidates are ranked by number of first-place rankings
  - Winner(s) is/are candidate(s) with the most first-place rankings
  - Method is used in many elections
    including many local and state elections in US and partly German Bundestag
- Antiplurality
  - Candidate with least last-place rankings wins
  - Candidates ranked from last to first by the number of last-place rankings they receive

Club president election example

- Anne (A), Brigitte (B), Claus (C), and David (D) running for president of a club
- club has 27 members
- 24 possible preference lists, but for this example only 4 are used

Preference list	**Number of occurrences**
A B C D	12
B C D A	7
C D A B	5
D C B A	3
Other preferences	0

- Who is the elected for president?

And the winner is ...

Preference list	Number of occurrences
A B C D	12
B C D A	7
C D A B	5
D C B A	3
Other preferences	0

- **Anne !**
  - Plurality
  - 12 (most) first-place votes
- **Claus !**
  - Antiplurality
  - No (least) last-place votes

Examples of social choice function

- **Instant runoff**
  - Candidate(s) with the least first-place rankings is/are removed
  - New set of preference lists for a smaller set of candidates
  - Repeated until all candidates are eliminated
  - Social choice is formed by listing candidates in reverse order in which they were eliminated
  - Used for elections in Australia and for presidential elections in Ireland.

And the winner is ...

Preference list	Number of occurrences
A B C D	12
B C D A	7
C D A B	5
D C B A	3
Other preferences	0

- Instant runoff
  - David eliminated first

Preference list	No. of occu.
A B C	12
B C A	7
C A B	5
C B A	3
Other preferences	0

- continued ...
  - Brigitte is eliminated second

Preference list	No. of occu.
A C	12
C A	7
C A	5
C A	3
Other preferences	0

  - Anne is emilinated last

⇒ **Claus !**

Examples of social choice functions

- **Borda count**
  - With k candidates
    - ⋆ k - 1 points are given for a first place ranking
    - ⋆ k - 2 points for a second place ranking
    - ⋆ and so on ...
  - Candidates ranked by total sum of points they receive
  - Candidate(s) with the most points win(s)
  - Method (or derivatives) used frequently for sports-related polls

And the winner is ...

Preference list	Number of occurrences
A B C D	12
B C D A	7
C D A B	5
D C B A	3
Other preferences	0

- Borda count
  - Anne: $(12 \times 3) + (5 \times 1) = 41$
  - Brigitte: $(12 \times 2) + (7 \times 3) + (3 \times 1) = 48$
  - Claus: $(12 \times 1) + (7 \times 1) + (5 \times 3) + (3 \times 2) = 47$
  - David: $(7 \times 1) + (5 \times 2) + (3 \times 3) = 26$

$\Rightarrow$ **Brigitte !**

Wait ...

- Anne won wrt. Plurality
- Claus won according to Antiplurality
- Claus won again wrt. Instant runoff
- Brigitte won wrt. Borda count

What?!? How ... ?!? 😱

- Three different winners using four methods?
- So winner is depending on voting method?
- **Does this seem reasonable?**

Condorcet

- Marquis Nicolas de Condorcet (1743–1794)
- French liberal thinker in the era of the French Revolution
- philosopher, mathematician, and political scientist
- Pursued by the revolutionary authorities for criticizing them
- Died in prison
- Essay on the Application of Analysis to the Probability of Majority Decisions (1785):

Essay sur l’Application de l’Analyse á la Probabilité des Décisions Rendue á la Pluralité des Voix

Condorcet's 2 prominent insights

- **Condorcet's jury theorem**
  - Each member of jury has chance of making a correct judgment on whether a defendant is guilty
    - ⋆ equal and independent
    - ⋆ better than random
    - ⋆ worse than perfect

⇒ majority of jurors is more likely to be correct than each individual juror

⇒ Probability of correct majority judgment approaches 1 as jury size increases

⇒ Under certain conditions, majority rule is good at ‘tracking the truth’

Condorcet's 2 prominent insights

- **Condorcet's paradox**

  Majority preferences can be 'irrational' (intransitive)

  – even when individual preferences are 'rational' (transitive).

  – Example

Preference list	**No. of occu.**
A B C	1 / 3
B C A	1 / 3
C A B	1 / 3

⇒ there are majorities (of two thirds)

- ⋆ for A against B
- ⋆ for B against C
- ⋆ for C against A

⇒ **Cycle** violates transitivity

Condorcet

- **Condorcet winner**
  Candidate who beat all other candidates in head-to-head contests
  - Examples
    - ⋆ No Condorcet winner in Condorcet's paradox
    - ⋆ Coleman in 1998 Minnesota governor election example
- **Condorcet loser**
  Candidate who loses to all other candidates in head-to-head contests

Condorcet winner criterion

> Whenever there is a Condorcet winner,
> that candidate is the unique winner of the election.

- Plurality does not satisfy Condorcet winner criterion

Ventura won, but

- 63% of voters liked him least!
- Coleman wins pairwise comparisons
  - 55% prefer Coleman to Humphrey
  - 63% prefer Coleman to Ventura

  - Coleman was the Condorcet winner
  - Ventura won

Condorcet winner criterion

Whenever there is a Condorcet winner,
that candidate is the unique winner of the election.

- Borda count does not satisfy Condorcet winner criterion
  - Exampel

Preference list	No. of occu.
A B C	3
B C A	2

  - Condorcet winner is A
  - Borda count
    - ⋆ A: $(2 \times 3) + (0 \times 1) = 6$
    - ⋆ B: $(1 \times 3) + (2 \times 2) = 7$
    - ⋆ C: $(0 \times 3) + (2 \times 1) = 2$

    $\Rightarrow$ **B is winner**

- Btw: Instant runoff does not either

Condorcet winner criterion

Whenever there is a Condorcet winner,
that candidate is the unique winner of the election.

- Plurality does not satisfy Condorcet winner criterion
- Borda count does not satisfy Condorcet winner criterion
- Instant runoff does not ...

- Are there any?
- Yes, there are!
- However, all of them run into other problems

What about other criteria?

Condorcet winner criterion example

- Choose winner based on head-to-head contests

⇒ Make sure Condorcet winner criterion is satisfied

- Example: Sequential pairwise voting
  - fix an (arbitrary) order of candidates
  - rounds of head-to-head contests between candidates following fixed order
  - winner of contest between the first two goes up against third candidate ...
  - until one candidate survives

- Satisfies the Condorcet winner criterion
  - Condorcet winner will beat everyone else on the list

Condorcet winner criterion example

Preference list	Number of occurrences
A B C D	12
B C D A	7
C D A B	5
D C B A	3
Other preferences	0

- Fixed ordering A B C D
  - 17 voters prefer A to B, only 10 B to A
  - ⇒ A beats B 17:10
  - C beats A 15:12
  - C beats D 15:12

  ⇒ C is the winner
- Fixed ordering A C B D ⇒ B is the winner
- Fixed ordering B C A D ⇒ D is the winner
- Fixed ordering B C D A ⇒ A is the winner

What?!? How ... ?!? Not good !!! 😱

Some formalism required

- Set $N = 1, 2, ..., n$ of individuals $(n \geqslant 2)$
- Set of social alternatives $X = x, y, z, ...$
- Each individual $i \in N$ has a preference ordering R_i over alternatives:
  - $\Rightarrow$ complete and transitive relation on X
- For any $x, y \in X$: xR_iy means that individual i prefers x to y
- xP_iy if xR_iy and not yR_ix ('individual i strictly prefers x to y')
- Profile

$$< R_1, R_2, ..., R_n >$$

combination of preference orderings across individuals

Some more formalism required

- Preference aggregation rule F
  - function that assigns to each profile a social preference relation $R = F(R_1, R_2, ..., R_n)$ on X

$$F : < R_1, R_2, ..., R_n > \; \rightarrow \; R = F(R_1, R_2, ..., R_n)$$

- Example: pairwise majority voting (Condorcet)
  - For any profile $< R_1, R_2, ..., R_n >$ and any $x, y \in X$:

    xRy if and only if at least as many individuals have xR_iy as have yR_ix

    or

$$|i \in N : xR_iy| \geqslant |i \in N : yR_ix|$$

Alternative criteria

- **Independence of irrelevant alternatives**

  Description

  – Candidate A is ranked higher than candidate B
  – Some voters change their preference lists, but no voter changes their preference between A and B
  ⇒ A should remain ranked higher than B

Societal preference between two candidates should depend only on the voters' preferences between A and B

  Mathematical formulation:

  – For any two profiles $< R_1, R_2, ..., R_n >$ and $< R_1^*, R_2^*, ..., R_n^* >$
  – For any $x, y \in X$
  – if for all $i \in N$:
  R_i's ranking between x and y coincides with R_i^*'s ranking between x and y

$$xRy \text{ if and only if } xR_y^*.$$

Alternative criteria

- **Independence of irrelevant alternatives**

Example: 1995 Women's Figure Skating World Championship

Ranking before last skater:

1. Chen Lu (China)
2. Nicole Bobek (US)
3. Suraya Bonaly (France)

last skater:
Michelle Kwan (US), who became 4.

Ranking after last skater:

1. Chen Lu (China)
2. Suraya Bonaly (France)
3. Nicole Bobek (US)
4. Michelle Kwan (US)

- Note: Nicole Bobek (US) and Suraya Bonaly (France) (2nd and 3rd before last skater) changed places!

Alternative criteria

- **Monotonicity**

  Description

  - Some voters move candidate A up in their preference lists
  - No voters move A down

  $\Rightarrow$ A cannot move down in the final ranking

  Mathematical formulation

  - For any profile $< R_1, R_2, ..., R_n >$ in the domain of F
  - Social preference relation R is complete and transitive

Restaurant type example

- 17 conference attandancees
- 4 suggestions for dinner restaurant type
- Selected method: Instant runoff

Preference list	Number of occurrences
Thai Chinese Italian German	6
Chinese Thai Italian German	5
Italian German Chinese Thai	4
German Italian Thai Chinese	2
Other preferences	0

- Which type of restaurant to choose for dinner?

And the winner is ...

Preference list	Number of occurrences
Thai Chinese Italian German	6
Chinese Thai Italian German	5
Italian German Chinese Thai	4
German Italian Thai Chinese	2
Other preferences	0

- Instant runoff:
  - German eliminated first
  - Chinese eliminated second
  - Italian eliminated last

  ⇒ Thai is the winner!

- Right before leaving, two voters from last row changed their mind
  - German Italian Thai Chinese
  - replaced by
  - German Thai Italian Chinese

And the winner is ...

Preference list	Number of occurrences
Thai Chinese Italian German	6
Chinese Thai Italian German	5
Italian German Chinese Thai	4
German Thai Italian Chinese	2
Other preferences	0

- Instant runoff:
  - German eliminated first
  - Italian eliminated second
  - Thai eliminated last

  ⇒ Chinese is the winner!

What?!?

Thai moved up in some preferences and went from winning to losing!

Arrow's list of conditions

- **Universal domain**
  - Voters can choose any possible preference order
  - The domain of F is the set of all logically possible profiles of complete and transitive individual preference orderings.
- **Ordering**
  This is monotonicity or ordering as discussed above
- **Independence of irrelevant alternatives**
  As discussed above

Arrow's list of conditions

- **Weak Pareto principle**
  - If all voters prefer x over y, this should hold for final ranking
  - For any profile $< R_1, R_2, ..., R_n >$ in the domain of F
  - If for all $i \in N$: xP_iy then xPy
- **Nondictatorship**
  - There should not be a dictator
  - One voter whose preference list determines the societal ranking completely.
  - There does not exist an individual $i \in N$ such that
    - ⋆ for all $< R_1, R_2, ..., R_n >$ in the domain of F
    - ⋆ for all $x, y \in X$
  - xP_iy implies xPy.

Main Result

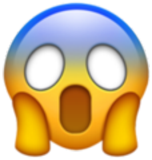
- Kenneth Joseph Arrow (1921 - 2017)
- American economist, mathematician, writer, and political theorist
- 1972 joint winner of the Nobel Memorial Prize in Economic Sciences (with John Hicks)
- Many of his former graduate students won the Nobel Memorial Prize themselves
- Most significnt contribution:

Arrow's impossibility theorem (1951)

If there are more than two candidates, then any social choice method cannot satisfy all of Arrow's five conditions.

Consequences and Implications

- All social choice methods have flaws
- Even most that are used for politcal elections throughout the world
- Also holds for most ranking methods
  - in sports
  - in games
  - etc



Consequences and Implications

- Weakening or relaxing conditions

  Works with different conditions and corresponding methods

- Example: independence of irrelevant alternatives
  - Intensity of voters' preference between two candidates
  - Number of other candidates listed between the two candidates
  - Intensity of binary independence criterion:
    - ⋆ If some voters change their preference lists
    - ⋆ No voter changes their preference between candidates A and B or the intensity of their preference

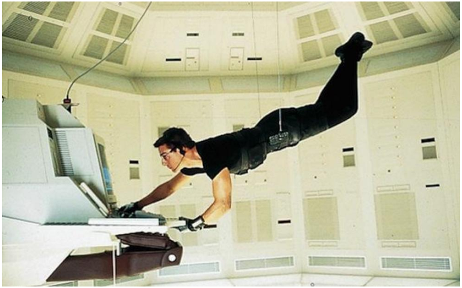
    $\Rightarrow$ Ranking of A and B in the social choice should not change

Borda count satisfies the conditions of Arrow's theorem with

- independence of irrelevant alternatives replaced by
- intensity of binary independence

Challenges for Ranking in Games

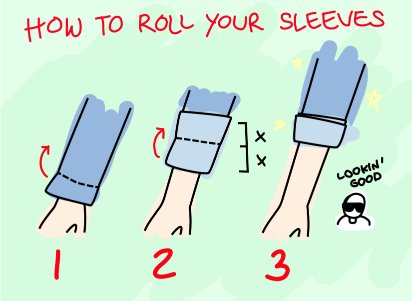
- Arrow's impossibility theorem
- Statistical Comparisons
- Choice of Fitness Functions
- Choice of Test Cases



Practical Recommendations

How to decide on a ranking method

- Rarity of criteria
- Perceived fairness
- Simplicity for transparency

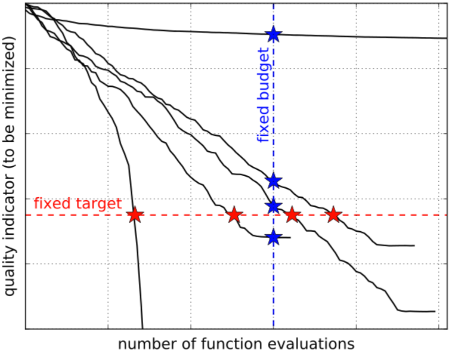


Overview of existing methods
`https://en.wikipedia.org/wiki/Comparison_of_electoral_systems`

Alternative: Mechanism Design

Lessons from EC Benchmarking

- Relevancy
- Fixed targets vs. fixed runtime
- Characteristics of problems (ELA)
- Evaluation Robustness (instances)
- Expected runtime measure
- Easy comparisons
- Info on optima?



Open Problems

- Selection of Ranking Mechanism (Which criteria can be relaxed?)
- Characterisation of problems (How do we guarantee completeness?)
- Long-term Ranking (How does ELO fit in?)
- Appropriate and practical noise handling
- Game Evaluation Measures

Games Benchmark

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