technische universität dortmund

Computational Intelligence

Winter Term 2021/22

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Plan for Today

- Deep Neural Networks
  - Model
  - Training
- Convolutional Neural Networks
  - Model
  - Training

Deep Neural Networks (DNN)

DNN = Neural Network with > 3 layers

we know: L = 3 layers in MLP sufficient to describe arbitrary sets

What can be achieved by more than 3 layers?

information stored in weights of edges of network

→ more layers → more neurons → more edges → more information storable

Which additional information storage is useful?

traditionally : handcrafted features fed into 3-layer perceptron

modern viewpoint : let L-k layers learn the feature map, last k layers separate!

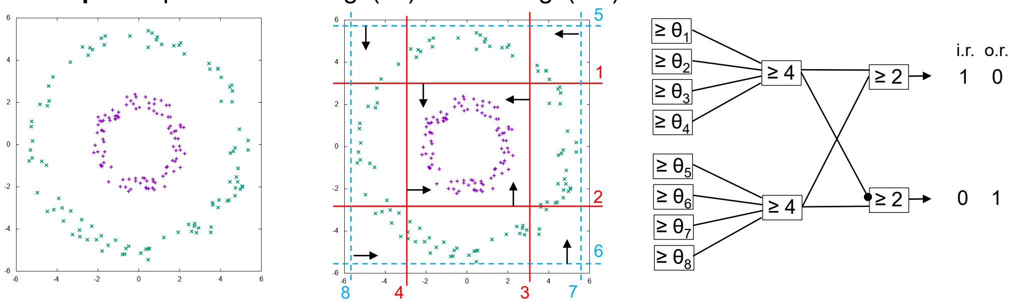
advantage:

human expert need not design features manually for each application domain

⇒ no expert needed, only observations!

Deep Neural Networks (DNN)

example: separate 'inner ring' (i.r.) / 'outer ring' (o.r.) / 'outside'



⇒ MLP with 3 layers and 12 neurons

Is there a simpler way?

observations $(x, y) \in \mathbb{R}^n \times \mathbb{B}$ feature map $F(x) = (F_1(x), \ldots, F_m(x)) \in \mathbb{R}^m$

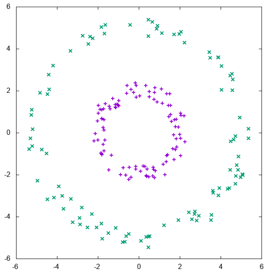
feature = measurable property of an observation or
numerical transformation of observed value(s)

⇒ find MLP on transformed data points (F(x), y)

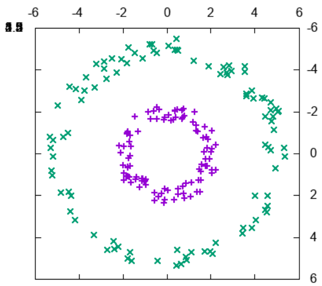
Deep Neural Networks (DNN)

example: separate 'inner ring' / 'outer ring'

- feature map $F(x) = (x_1, x_2, \sqrt{x_1^2 + x_2^2}) \in \mathbb{R}^3$

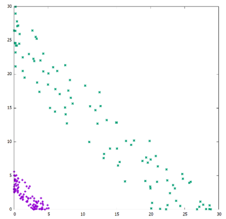


2D → 3D



- feature map $F(x) = (x_1^2, x_2^2) \in \mathbb{R}^2$

x_1^2, x_2^2 → $\boxed{\theta \geq 9}$ → $\begin{cases} 1\text{: outer} \\ 0\text{: inner} \end{cases}$



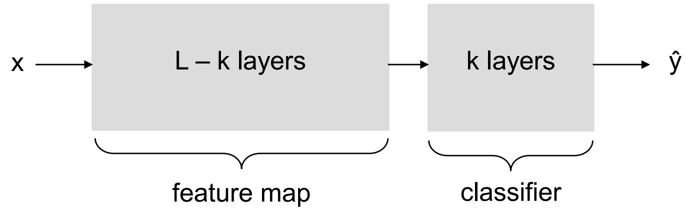
Deep Neural Networks (DNN)

but: how to find useful features?

→ typically designed by experts with domain knowledge

→ traditional approach in classification:
1. design & select appropriate features
2. map data to feature space
3. apply classification method to data in feature space

modern approach via DNN: learn feature map and classification simultaneously!



proven: MLP can approximate any continuous map with aribitrary accuracy

Deep Multi-Layer Perceptrons

contra:
- danger: overfitting
  → need larger training set (expensive!)
  → optimization needs more time
- response landscape changes
  → more sigmoidal activiations
  → gradient vanishes
  → small progress in learning weights

countermeasures:
- regularization / dropout
  → data augmentation
  → parallel hardware (multi-core / GPU)
- not necessarily bad
  → change activation functions
  → gradient does not vanish
  → progress in learning weights

vanishing gradient: (underlying principle)

forward pass $y = f_3(f_2(f_1(x; w_1); w_2); w_3)$ $\quad$ f_i ≈ activation function

backward pass $(f_3(f_2(f_1(x; w_1); w_2); w_3))' =$
$f_3'(f_2(f_1(x;w_1);w_2);w_3) \cdot f_2'(f_1(x;w_1);w_2) \cdot f_1'(x;w_1)$ ***chain rule!***

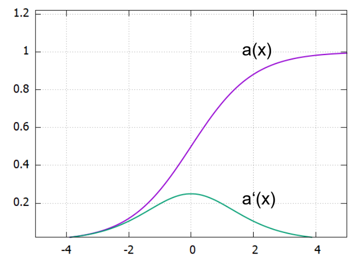
→ repeated multiplication of values in (0,1) → 0

Deep Multi-Layer Perceptrons

vanishing gradient: $a(x) = \dfrac{e^x}{e^x+1} = \dfrac{1}{1+e^{-x}} \quad \to \quad a'(x) = a(x)\cdot(1-a(x))$

$\forall x \in \mathbb{R}: \quad \boxed{a(x)\cdot(1-a(x)) \leq \frac{1}{4}} \quad \Leftrightarrow \quad \left(a(x) - \frac{1}{2}\right)^2 \geq 0$ ☑

⇒ gradient $a'(x) \in [0, \frac{1}{4}]$



<u>principally:</u> desired property in learning process!
if weights stabilize such that neuron almost always either fires [i.e., a(x) ≈ 1] or not fires [i.e., a(x) ≈ 0] then gradient ≈ 0 and the weights are hardly changed

⇒ leads to convergence in the learning process!

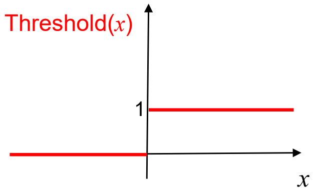
while learning, updates of weights via partial derivatives:

$$\frac{\partial f(w,u;x,z^*)}{\partial w_{ij}} = 2\sum_{k=1}^{K}[a(u_k'y) - z_k^*] \cdot \underbrace{a'(u_k'y)}_{\leq \frac{1}{4}} \cdot u_{jk} \cdot \underbrace{a'(w_j'x)}_{\leq \frac{1}{4}} \cdot x_i \qquad \text{(L= 2 layers)}$$

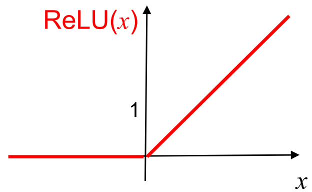
⇒ in general $f_{w_{ij}} = O(4^{-L}) \to 0$ as $L\uparrow$ $\qquad L \leq 3$: effect neglectable; but $L \gg 3$ ☒

non-sigmoid activation functions

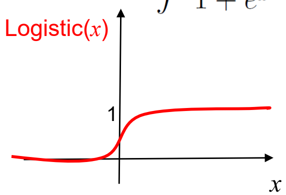
$$\int \mathbb{1}_{[x \geq 0]}(x)\,dx = \left\{ \begin{array}{ll} 0 & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{array} \right\} = \max\{0, x\} = \text{ReLU}(x)$$



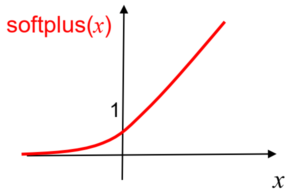
$\Rightarrow$



$$\int \frac{e^x}{1+e^x}\,dx = \log(1+e^x) = \text{softplus}(x)$$



$\Rightarrow$



dropout

- applied for regularization (against overfitting)
- can be interpreted as inexpensive approximation of **bagging**

↓

aka: bootstrap aggregating, model averaging, ensemble methods

create k training sets by drawing with replacement
train k models (with own exclusive training set)
combine k outcomes from k models (e.g. majority voting)

- parts of network is effectively switched off
  e.g. multiplication of outputs with 0,
  e.g. use inputs with prob. 0.8 and inner neurons with prob. 0.5
- gradient descent on switching parts of network
  → artificial perturbation of greediness during gradient descent
- can reduce computational complexity if implemented sophistically

data augmentation (counteracts overfitting)

→ extending training set by slightly perturbed true training examples

- best applicable if inputs are **images**: translate, rotate, add noise, resize, …


original image


rotated


resized


noisy


noisy + rotated

- if x is **real vector** then adding e.g. small gaussian noise
  → here, utility disputable (artificial sample may cross true separating line)

extra costs for acquiring additional annotated data are **inevitable**!

stochastic gradient descent

- partitioning of training set B into **(mini-) batches** of size b

traditionally: 2 extreme cases		now:
update of weights		update of weights
- after each training example	b = 1	- after b training examples
- after all training examples	b = |B|	where 1 < b < |B|

- search in subspaces → counteracts greediness → better generalization
- accelerates optimization methods (parallelism possible)

choice of batch size b
b large ⇒ better approximation of gradient
b small ⇒ better generalization

b also depends on available hardware
b too small ⇒ multi-cores underemployed

often b ≈ 100 (empirically)

cost functions

- *regression*

  N training samples (x_i, y_i)

  insist that $f(x_i;\ \theta) = y_i$ for i=1,…, N

  if $f(x; \theta)$ linear in θ then $\theta^T x_i = y_i$ for i=1,…, N or $X\,\theta = y$

  $\Rightarrow$ best choice for θ: least square estimator (LSE)

  $\Rightarrow (X\,\theta - y)^T (X\,\theta - y) \ \to \min\limits_{\theta}!$

  in case of MLP: $f(x; \theta)$ is <u>nonlinear</u> in θ

  $\Rightarrow$ best choice for θ: (nonlinear) least square estimator; aka TSSE

  $\Rightarrow \sum\limits_i (f(x_i; \theta) - y_i)^2 \to \min\limits_{\theta}!$

cost functions

- *classification*

  N training samples (x_i, y_i) where $y_i \in \{ 1, …, C \}$, C = #classes

  → want to estimate probability of different outcomes for unknown sample

  → decision rule: choose class with highest probability (given the data)

  <u>idea</u>: use maximum likelihood estimator (MLE)

  = estimate unknown parameter θ such that likelihood of sample $x_1, …, x_N$ gets maximal as a function of θ

  <u>likelihood function</u>

$$L(\theta; x_1, \ldots, x_N) := f_{X_1,\ldots,X_N}(x_1, \ldots, x_N; \theta) = \prod_{i=1}^{N} f_X(x_i; \theta) \to \max_{\theta}!$$

here: random variable $X \in \{1, …, C\}$ with $P\{ X = i \} = q_i$ (true, but unknown)

→ we use relative frequencies of training set $x_1, …, x_N$ as estimator of q_i

$$\hat{q}_i = \frac{1}{N} \sum_{j=1}^{N} \mathbb{1}_{[x_j = i]} \quad \Rightarrow \text{there are } N \cdot \hat{q}_i \text{ samples of class } i \text{ in training set}$$

⇒ the neural network should output $\hat{p}$ as close as possible to $\hat{q}$! [actually: to q]

likelihood $L(\hat{p}; x_1, \ldots, x_N) = \prod\limits_{k=1}^{N} P\{X_k = x_k\} = \prod\limits_{i=1}^{C} \hat{p}_i^{N \cdot \hat{q}_i} \to \max!$

$$\log L = \log \left(\prod_{i=1}^{C} \hat{p}_i^{N \cdot \hat{q}_i} \right) = \sum_{i=1}^{C} \log \hat{p}_i^{N \cdot \hat{q}_i} = N \underbrace{\sum_{i=1}^{C} \hat{q}_i \cdot \log \hat{p}_i}_{-H(\hat{q}, \hat{p})} \to \max!$$

⇒ maximizing $\log L$ leads to same solution as minimizing **cross-entropy** $H(\hat{q}, \hat{p})$

in case of *classification*

use softmax function $P\{y = j \,|\, x\} = \dfrac{e^{w_j^T x + b_j}}{\sum_{i=1}^{C} e^{w_i^T x + b_i}}$ in output layer

→ multiclass classification: probability of membership to class j = 1, …, C

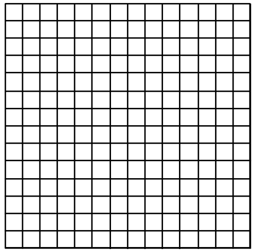
→ class with maximum excitation w'x+b has maximum probabilty

→ decision rule: element x is assigned to class with maximum probability most often used in graphical applications (2-D input; also possible: k-D tensors)

layer of CNN = 3 stages

1. convolution
2. nonlinear activation (e.g. ReLU)
3. pooling

I(x,y)



K(i,j)

$$\begin{array}{cccc} -\delta & & \delta & \\ \begin{array}{|c|c|c|}\hline -1 & -2 & -1 \\ \hline 1 & 1 & 1 \\ \hline -2 & 1 & -2 \\ \hline \end{array} & & & \begin{array}{c} -\delta \\ \\ \delta \end{array} \end{array}$$

example

1. Convolution

local filter / kernel K(i, j) applied to each cell of image I(x, y)

$$S(x,y) = (K * I)(x,y) = \sum_{i=-\delta}^{\delta} \sum_{j=-\delta}^{\delta} I(x-i, y-j) \cdot K(i,j)$$

example: edge detection with Sobel kernel

→ two convolutions

$K_x = \begin{pmatrix} -1, 0, 1 \\ -2, 0, 2 \\ -1, 0, 1 \end{pmatrix}$ yields S_x

$K_y = \begin{pmatrix} -1, -2, -1 \\ 0, 0, 0 \\ 1, 2, 1 \end{pmatrix}$ yields S_y

$$S(x,y) = \sqrt{S_x(x,y)^2 + S_y(x,y)^2}$$

original image I(x,y)

image S(x,y) after convolution

filter / kernel

well known in image processing; typically hand-crafted!

here: values of filter matrix learnt in CNN !

actually: many filters active in CNN

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 \end{pmatrix}$$

e.g. horizontal line detection

stride

= distance between two applications of a filter (horizontal s_h / vertical s_v)

→ leads to smaller images if s_h or s_v > 1

padding

= treatment of border cells if filter does not fit in image

- “valid“ : apply only to cells for which filter fits → leads to smaller images
- “same“ : add rows/columns with zero cells; apply filter to all cells (→ same size)

2. nonlinear activation

$a(x) = \text{ReLU}(x^T W + c)$

3. pooling

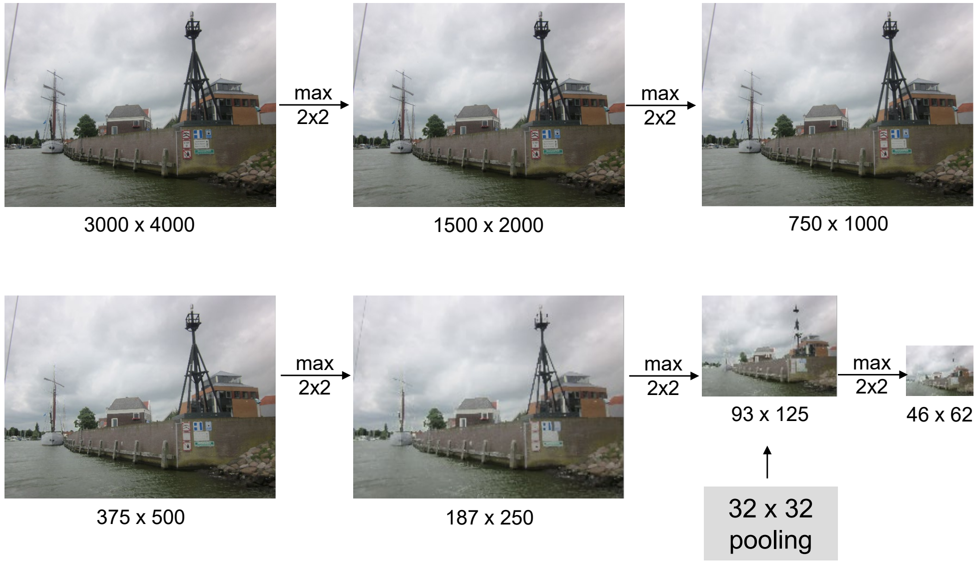
in principle: summarizing statistic of nearby outputs

e.g. **max-pooling** m(i,j) = max(I(i+a, j+b) : a,b = -δ, …, 0, … δ) for δ > 0

- also possible: mean, median, matrix norm, …

- can be used to reduce matrix / output dimensions

example: max-pooling 2x2 (iterated), stride = 2

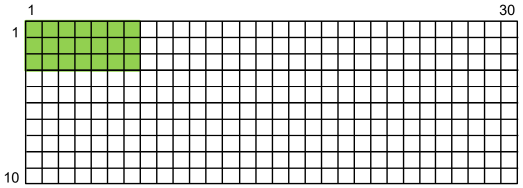


Pooling with Stride

c_{in} : columns of input
r_{in} : rows of input
f_c : columns of filter
f_r : rows of filter
s_c : stride for columns
s_r : stride for rows

image size : $r_{in} \times c_{in}$
filter size : $f_r \times f_c$

<u>assumptions:</u>
$f_c \leq c_{in}$
$f_r \leq f_{in}$
`padding = valid`



How often fits the filter in image horizontally?

$pos_1 = 1$
$pos_2 = pos_1 + s_c$
$pos_3 = pos_2 + s_c = (pos_1 + s_c) + s_c = pos_1 + 2 \cdot s_c$
$\vdots$
$pos_k = pos_1 + (k-1) \cdot s_c$

thus, find largest k such that

$$pos_1 + (k-1) \cdot s_c + (f_c - 1) \leq c_{in}$$
$$\Leftrightarrow \quad (k-1) \cdot s_c + f_c \leq c_{in}$$
$$\Leftrightarrow \quad k \leq (c_{in} - f_c) / s_c + 1 \qquad \text{(integer division!)}$$
$$\Rightarrow \quad k = \left\lfloor \frac{c_{in} - f_c}{s_c} \right\rfloor + 1 = c_{out}$$

[analog reasoning for rows!]

CNN architecture:

- several consecutive convolution layers (also parallel streams); possibly dropouts
- flatten layer (→ converts k-D matrix to 1-D matrix required for MLP input layer)
- fully connected MLP

examples: