technische universität dortmund

# Computational Intelligence

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## Plan for Today — Lecture 08

- Approximate Reasoning
- Fuzzy Control



## Approximative Reasoning — Lecture 08

**So far:**

- p: IF *X* is A THEN *Y* is B

  → R(x, y) = Imp( A(x), B(y) )  — rule as relation; fuzzy implication

- rule: IF *X* is A THEN *Y* is B
  fact: *X* is A‘
  conclusion: *Y* is B‘

  → $B'(y) = \sup_{x \in X} t( A'(x), R(x, y) )$  — composition rule of inference

**Thus:**

- $B'(y) = \sup_{x \in X} t( A'(x), Imp( A(x), B(y) ) )$

| | |
|---|---|
| given | : fuzzy rule |
| input | : fuzzy set A‘ |
| output | : fuzzy set B‘ |



## Approximative Reasoning — Lecture 08

**here:**

$$A'(x) = \begin{cases} 1 & \text{for } x = x_0 \\ 0 & \text{otherwise} \end{cases}$$

crisp input!

$$B'(y) = \sup_{x \in X} t( A'(x), Imp( A(x), B(y) ) )$$

$$= \begin{cases} \sup_{x \neq x_0} t( 0, Imp( A(x), B(y) ) ) & \text{for } x \neq x_0 \\ t( 1, Imp( A(x_0), B(y) ) ) & \text{for } x = x_0 \end{cases}$$

$$= \begin{cases} 0 & \text{for } x \neq x_0 & \text{since } t(0, a) = 0 \\ Imp( A(x_0), B(y) ) & \text{for } x = x_0 & \text{since } t(a, 1) = a \end{cases}$$

**Lemma:**

a) $t(a, 1) = a$

b) $t(a, b) \le \min \{ a, b \}$

c) $t(0, a) = 0$

***Proof:***

ad a) Identical to axiom 1 of t-norms.

ad b) From monotonicity (axiom 2) follows for $b \le 1$, that $t(a, b) \le t(a, 1) = a$ (by a). Commutativity (axiom 3) and monotonicity lead in case of $a \le 1$ to $t(a, b) = t(b, a) \le t(b, 1) = b$. Thus, $t(a, b)$ is less than or equal to a as well as b, which in turn implies $t(a, b) \le \min\{ a, b \}$.

ad c) From b) follows $0 \le t(0, a) \le \min \{ 0, a \} = 0$ and therefore $t(0, a) = 0$. ■





**Multiple rules:**

IF X is $A_1$, THEN Y is $B_1$ → $R_1(x, y) = Imp_1( A_1(x), B_1(y) )$
IF X is $A_2$, THEN Y is $B_2$ → $R_2(x, y) = Imp_2( A_2(x), B_2(y) )$
IF X is $A_3$, THEN Y is $B_3$ → $R_3(x, y) = Imp_3( A_3(x), B_3(y) )$
… …
IF X is $A_n$, THEN Y is $B_n$ → $R_n(x, y) = Imp_n( A_n(x), B_n(y) )$
X is A‘

---

Y is B‘

**Multiple rules for <u>crisp input</u>:** $x_0$ is given

$B_1‘(y) = Imp_1(A_1(x_0), B_1(y) )$
…
$B_n‘(y) = Imp_n(A_n(x_0), B_n(y) )$

} aggregation of rules or local inferences necessary!

**aggregate!** ⇒ $B‘(y) =$ **aggr**$\{ B_1‘(y), \ldots, B_n‘(y) \}$, where **aggr** = { min, max





<u>**FITA**: “**F**irst **i**nference, **t**hen **a**ggregate!“</u>

1. Each rule of the form IF *X* is $A_k$ THEN *Y* is $B_k$ must be transformed by an appropriate fuzzy implication $Imp_k(\cdot,\cdot)$ to a relation $R_k$ : $R_k(x, y) = Imp_k( A_k(x), B_k(y) )$.
2. Determine $B_k‘(y) = R_k(x, y) \circ A‘(x)$ for all $k = 1, \ldots, n$ (local inference).
3. Aggregate to $B‘(y) = \beta( B_1‘(y), \ldots, B_n‘(y) )$.

<u>**FATI**: “**F**irst **a**ggregate, **t**hen **i**nference!“</u>

1. Each rule of the form IF *X* ist $A_k$ THEN *Y* ist $B_k$ must be transformed by an appropriate fuzzy implication $Imp_k(\cdot, \cdot)$ to a relation $R_k$ : $R_k(x, y) = Imp_k( A_k(x), B_k(y) )$.
2. Aggregate $R_1, \ldots, R_n$ to a **superrelation** with aggregating function **$\alpha(\cdot)$:** $R(x, y) = \alpha( R_1(x, y), \ldots, R_n(x, y) )$.
3. Determine $B‘(y) = R(x, y) \circ A‘(x)$ w.r.t. superrelation (inference).





**1. Which principle is better? FITA or FATI?**

**2. Equivalence of FITA and FATI ?**

**FITA:** $B‘(y) = \beta( B_1‘(y), \ldots, B_n‘(y) )$
$= \beta( R_1(x, y) \circ A‘(x), \ldots, R_n(x, y) \circ A‘(x) )$

**FATI:** $B‘(y) = R(x, y) \circ A‘(x)$
$= \alpha( R_1(x, y), \ldots, R_n(x, y) ) \circ A‘(x)$

**special case:**

$$A'(x) = \begin{cases} 1 & \text{for } x = x_0 \\ 0 & \text{otherwise} \end{cases}$$

crisp input!

**On the equivalence of FITA and FATI:**

**FITA:** $B'(y) = \beta(\, B_1'(y), \ldots, B_n'(y)\,)$

$= \beta(\, Imp_1(A_1(x_0), B_1(y)\,), \ldots, Imp_n(A_n(x_0), B_n(y)\,)\,)$

**FATI:** $B'(y) = R(x, y) \circ A'(x)$

$= \sup_{x \in X} t(\, A'(x), R(x, y)\,)$ (from now: special case)

$= R(x_0, y)$

$= \alpha(\, Imp_1(\, A_1(x_0), B_1(y)\,), \ldots, Imp_n(\, A_n(x_0), B_n(y)\,)\,)$

**evidently:** sup-t-composition with arbitrary t-norm and **$\alpha(\cdot) = \beta(\cdot)$**





- **AND-connected premises**

IF $X_1 = A_{11}$ AND $X_2 = A_{12}$ AND … AND $X_m = A_{1m}$ THEN $Y = B_1$
…
IF $X_n = A_{n1}$ AND $X_2 = A_{n2}$ AND … AND $X_m = A_{nm}$ THEN $Y = B_n$

reduce to single premise for each rule k:

$A_k(x_1,\ldots, x_m) = \min \{\, A_{k1}(x_1), A_{k2}(x_2), \ldots, A_{km}(x_m)\,\}$ or in general: t-norm

- **OR-connected premises**

IF $X_1 = A_{11}$ OR $X_2 = A_{12}$ OR … OR $X_m = A_{1m}$ THEN $Y = B_1$
…
IF $X_n = A_{n1}$ OR $X_2 = A_{n2}$ OR … OR $X_m = A_{nm}$ THEN $Y = B_n$

reduce to single premise for each rule k:

$A_k(x_1,\ldots, x_m) = \max \{\, A_{k1}(x_1), A_{k2}(x_2), \ldots, A_{km}(x_m)\,\}$ or in general: s-norm





**important:**

- if rules of the form **IF *X* is A THEN *Y* is B** interpreted as logical implication

  $\Rightarrow$ $R(x, y) = Imp(\, A(x), B(y)\,)$ makes sense

- we obtain: $B'(y) = \sup_{x \in X} t(\, A'(x), R(x, y)\,)$

$\Rightarrow$ the worse the match of premise $A'(x)$, the larger is the fuzzy set $B'(y)$

$\Rightarrow$ follows immediately from axiom 1: $a \le b$ implies $Imp(a, z) \ge Imp(b, z)$

**interpretation of output set B'(y):**

- $B'(y)$ is the set of values that are still possible
- each rule leads to an additional restriction of the values that are still possible

$\Rightarrow$ resulting fuzzy sets $B'_k(y)$ obtained from single rules must be mutually intersected!

$\Rightarrow$ aggregation via $B'(y) = \mathbf{min}\,\{\, B_1'(y), \ldots, B_n'(y)\,\}$





**important:**

- if rules of the form **IF *X* is A THEN *Y* is B** are not interpreted as logical implications, then the function Fct(·) in

$$R(x, y) = Fct(\, A(x), B(y)\,)$$

can be chosen as required for desired interpretation.

- frequent choice (especially in fuzzy control):
  - $R(x, y) = \min \{\, A(x), B(x)\,\}$ Mamdani – "implication"
  - $R(x, y) = A(x) \cdot B(x)$ Larsen – "implication"

$\Rightarrow$ of course, they are no implications but specific t-norms!

$\Rightarrow$ thus, if relation R(x, y) is given, then the *composition rule of inference*

$$B'(y) = A'(x) \circ R(x, y) = \sup_{x \in X} \min \{\, A'(x), R(x, y)\,\}$$

still can lead to a conclusion via fuzzy logic.

**example:** [JM96, S. 244ff.]

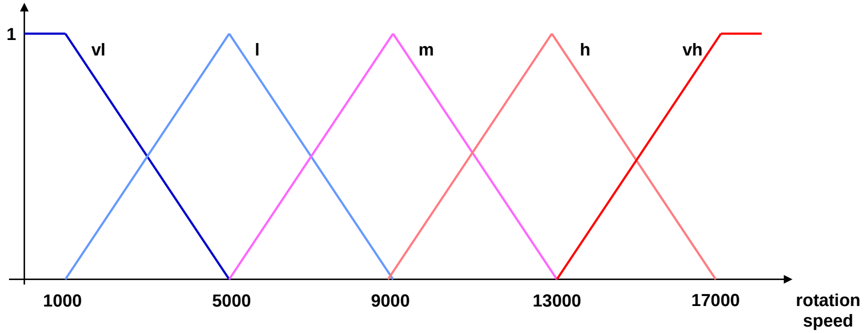
industrial drill machine → control of cooling supply

modelling

linguistic variable : **rotation speed**

linguistic terms : ***very low***, ***low***, ***medium***, ***high***, ***very high***

ground set : X with $0 \le x \le 18000$ [rpm]








**example:** (continued)

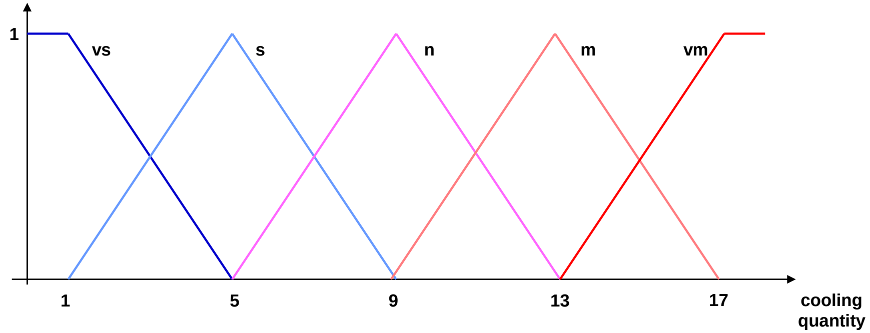
industrial drill machine → control of cooling supply

modelling

linguistic variable : **cooling quantity**

linguistic terms : ***very small***, ***small***, ***normal***, ***much***, ***very much***

ground set : Y with $0 \le y \le 18$ [liter / time unit]








**example:** (continued)

industrial drill machine → control of cooling supply

rule base

| | | | |
|---|---|---|---|
| **IF rotation speed IS** | ***very low*** | **THEN cooling quantity IS** | ***very small*** |
| | ***low*** | | ***small*** |
| | ***medium*** | | ***normal*** |
| | ***high*** | | ***much*** |
| | ***very high*** | | ***very much*** |

↑ sets $S_{vl}, S_l, S_m, S_h, S_{vh}$ — "rotation speed"

↑ sets $C_{vs}, C_s, C_n, C_m, C_{vm}$ — "cooling quantity"





**example:** (continued)

industrial drill machine → control of cooling supply

1. **input**: crisp value $x_0 = 10000 \text{ min}^{-1}$ (not a fuzzy set!)
   → **fuzzyfication** = determine membership for each fuzzy set over X
   → yields $S' = (0, 0, ¾, ¼, 0)$ via $x \; \alpha \; ( S_{vl}(x_0), S_l(x_0), S_m(x_0), S_h(x_0), S_{vh}(x_0) )$

2. FITA: locale **inference** ⇒ since Imp(0,a) = 0 we only need to consider:
   $S_m$: $C'_n(y) = \text{Imp}( ¾, C_n(y) )$
   $S_h$: $C'_m(y) = \text{Imp}( ¼, C_m(y) )$

3. **aggregation**:
   $C'(y) = $ **aggr** $\{ C'_n(y), C'_m(y) \} = \max\{ \text{Imp}( ¾, C_n(y) ), \text{Imp}( ¼, C_m(y) ) \}$

   ? ?

**example:** (continued)

industrial drill machine → control of cooling supply

$C'(y) = \max \{ \text{Imp}(\frac{3}{4}, C_n(y)), \text{Imp}(\frac{1}{4}, C_m(y)) \}$

in case of <u>control task</u> typically no logic-based interpretation:

→ max-aggregation and

→ relation R(x,y) not interpreted as implication.

often: R(x,y) = min(a, b) „Mamdani controller“

**thus:**

$C'(y) = \max \{ \min \{ \frac{3}{4}, C_n(y) \}, \min \{ \frac{1}{4}, C_m(y) \} \}$

→ graphical illustration

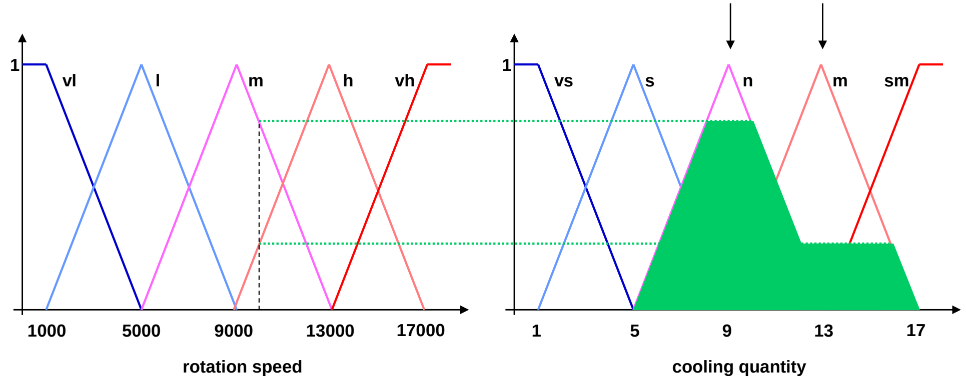




**example:** (continued)

industrial drill machine → control of cooling supply

$C'(y) = \max \{ \min \{ \frac{3}{4}, C_n(y) \}, \min \{ \frac{1}{4}, C_m(y) \} \}, x_0 = 10000$ [rpm]








**open and closed loop control:**

affect the dynamical behavior of a system in a desired manner

- **open loop control**

  control is aware of reference values and has a model of the system
  ⇒ control values can be adjusted,
  such that process value of system is equal to reference value

  problem: noise! ⇒ deviation from reference value not detected

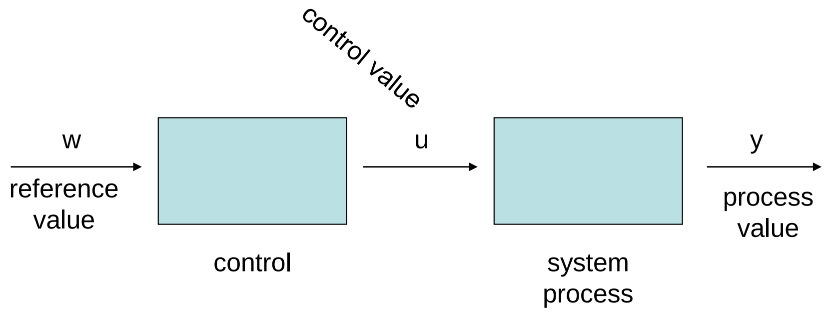
- **closed loop control**

  now: detection of deviations from reference value possible
  (by means of measurements / sensors)
  and new control values can take into account the amount of deviation



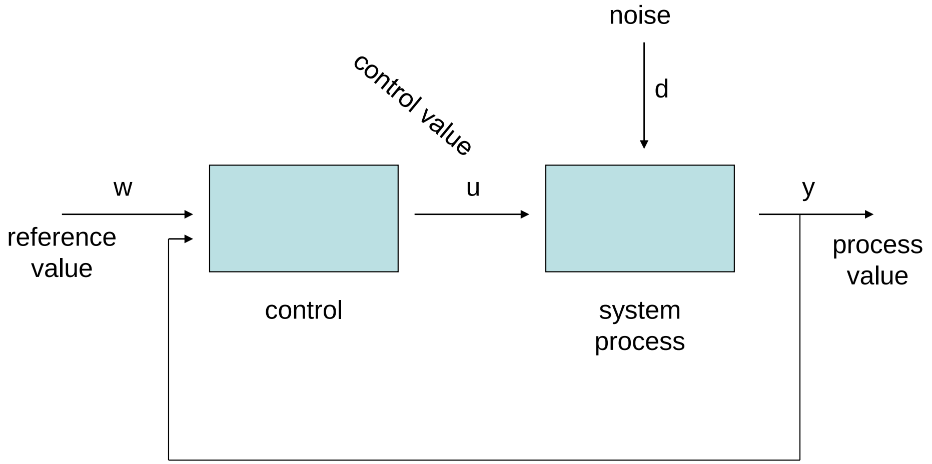


**open loop control**




assumption: undisturbed operation ⇒ process value = reference value

## closed loop control




control deviation = reference value – process value





## required:

model of system / process

→ as differential equations or difference equations (DEs)

→ well developed theory available

## so, why fuzzy control?

- there exists no process model in form of DEs etc. (operator/human being has realized control by hand)
- process with high-dimensional nonlinearities → no classic methods available
- control goals are vaguely formulated („soft“ changing gears in cars)





## fuzzy description of control behavior

IF X is $A_1$, THEN Y is $B_1$
IF X is $A_2$, THEN Y is $B_2$
IF X is $A_3$, THEN Y is $B_3$
…
IF X is $A_n$, THEN Y is $B_n$
X is A‘
_______________
Y is B‘

} similar to approximative reasoning

but fact A‘ is not a fuzzy set but a crisp input

→ actually, it is the current process value

fuzzy controller executes inference step

→ yields fuzzy output set B‘(y)

but crisp control value required for the process / system

→ defuzzification (= “condense” fuzzy set to crisp value)



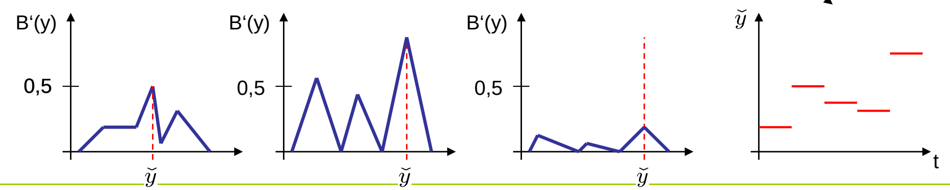


## defuzzification

**Def**: rule k active ⇔ $A_k(x_0) > 0$

- maximum method
  - only active rule with largest activation level is taken into account
    - → suitable for pattern recognition / classification
    - → decision for a single alternative among finitely many alternatives
  - selection independent from activation level of rule (0.05 vs. 0.95)
  - if used for control: incontinuous curve of output values (leaps)

$$\breve{y} = \text{argmax } B'(y)$$

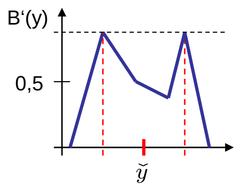


## defuzzification

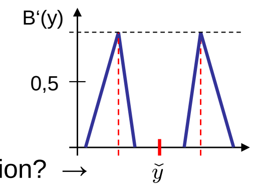
$Y^* = \{ y \in Y: B'(y) = hgt(B') \}$

- maximum mean value method
  - all active rules with largest activation level are taken into account
    - → interpolations possible, but need not be useful
    - → obviously, only useful for neighboring rules with max. activation
  - selection independent from activation level of rule (0.05 vs. 0.95)
  - if used in control: incontinuous curve of output values (leaps)

$$\breve{y} = \frac{1}{|Y^*|} \sum_{y^* \in Y^*} y^*$$




useful solution? →





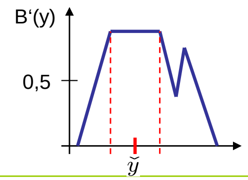


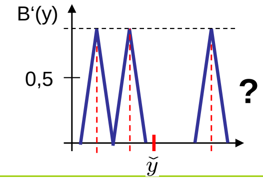

## defuzzification

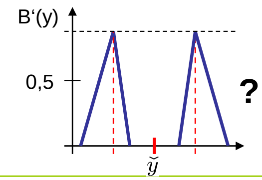
$Y^* = \{ y \in Y: B'(y) = hgt(B') \}$

- center-of-maxima method (COM)
  - only **extreme** active rules with largest activation level are taken into account
    - → interpolations possible, but need not be useful
    - → obviously, only useful for neighboring rules with max. activation level
  - selection indepependent from activation level of rule (0.05 vs. 0.95)
  - in case of control: incontinuous curve of output values (leaps)

$$\breve{y} = \frac{\inf Y^* + \sup Y^*}{2}$$








## defuzzification

- Center of Gravity (COG)
  - all active rules are taken into account
    - → but numerically expensive … …only valid for HW solution, today!
    - → borders cannot appear in output ( ∃ work-around )
  - if only single active rule: independent from activation level
  - continuous curve for output values

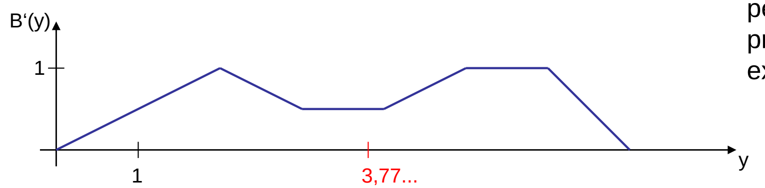
$$\breve{y} = \frac{\int y \cdot B'(y)\, dy}{\int B'(y)\, dy}$$





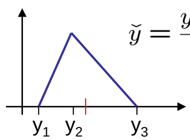
**Excursion: COG**

$$\breve{y} = \frac{\int y \cdot B'(y)\, dy}{\int B'(y)\, dy}$$



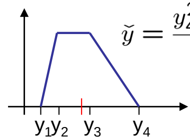

pendant in probability theory: expectation value

triangle:

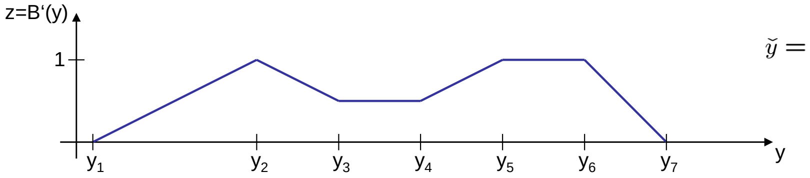



$$\breve{y} = \frac{y_1 + y_2 + y_3}{3}$$

trapezoid:




$$\breve{y} = \frac{y_4^2 + y_3^2 - y_2^2 - y_1^2 + y_3 y_4 - y_1 y_2}{3\,(y_4 + y_3 - y_2 - y_1)}$$




$$\breve{y} = \frac{\int y \cdot B'(y)\,dy}{\int B'(y)\,dy}$$

<u>assumption:</u> fuzzy membership functions piecewise linear

output set B‘(y) represented by sequence of points $(y_1, z_1)$, $(y_2, z_2)$, …, $(y_n, z_n)$

$\Rightarrow$ area under B‘(y) and weighted area can be determined additively piece by piece

$\Rightarrow$ linear equation z = m y + b $\Rightarrow$ insert $(y_i, z_i)$ and $(y_{i+1}, z_{i+1})$

$\Rightarrow$ yields m and b for each of the n-1 linear sections

$$\Rightarrow F_i = \int_{y_i}^{y_{i+1}} (m\,y + b)\,dy = \frac{m}{2}(y_{i+1}^2 - y_i^2) + b(y_{i+1} - y_i)$$

$$\Rightarrow G_i = \int_{y_i}^{y_{i+1}} y\,(m\,y + b)\,dy = \frac{m}{3}(y_{i+1}^3 - y_i^3) + \frac{b}{2}(y_{i+1}^2 - y_i^2)$$

$$\breve{y} = \frac{\sum_i G_i}{\sum_i F_i}$$





## Defuzzification

- Center of Area (COA)
  - developed as an approximation of COG
  - let $\hat{y}_k$ be the COGs of the output sets $B'_k(y)$:

$$\breve{y} = \frac{\sum_k A_k(x_0) \cdot \hat{y}_k}{\sum_k A_k(x_0)}$$

**how to:**
assume that fuzzy sets $A_k(x)$ and $B_k(x)$ are triangles or trapezoids
let $x_0$ be the crisp input value
for each fuzzy rule “IF $A_k$ is X THEN $B_k$ is Y“
determine $B'_k(y) = R(A_k(x_0), B_k(y))$, where R(.,.) is the relation
find $\hat{y}_k$ as center of gravity of $B'_k(y)$