THE VIRTUES OF METAHEURISTICS IN STOCHASTIC PROGRAMMING

Günter Rudolph

Department of Computer Science, TU Dortmund University, Germany, guenter.rudolph@tu-dortmund.de

1. Introduction

A metaheuristic can be regarded as an abstract, general strategy which guides other heuristics to search for good and hopefully optimal feasible solutions of difficult optimization problems. Only if the metaheuristic is instantiated with concrete rules, methods and parameters, then it becomes an optimization algorithm. As a consequence, a large variety of instances of a single metaheuristic are possible. Examples are tabu search, evolutionary algorithms and other stochastic local search approaches [1,6,7,8]. Here, we focus on continuous and mixed-integer optimization problems. Similar to deterministic direct search methods [10] these metaheuristics are appealing for solving real-world problems as they work without any derivatives under mildest conditions. Their main field of deployment has been deterministic static optimizations problems under single or multiple objective functions. But they have been shown to work also under noise in the objective function [2] and dynamic problems [5] with time-dependent parameters.

As soon as some parameters of a static, deterministic optimization problem are represented by random variables, then we encounter a stochastic programming problem [11]. Since the parameters are stochastic, the objective function value is a random variable. Needless to say, it is mathematically senseless to aim at optimizing a random quantity. Therefore we need another notion of optimality as in the deterministic context. This eventually leads to deterministic surrogate models. For example, if the objective function $f(x, \omega)$ has some random parameters $\omega \in \Omega$, a common deterministic surrogate model is the expectation model

$$E[\ f(x, \omega)\ ] \rightarrow \min!$$

or even bicriteria surrogate models that also take the variance into account:

$$\begin{aligned} E[\ f(x, \omega)\ ] &\rightarrow \min! \\ V[\ f(x, \omega)\ ] &\rightarrow \min! \end{aligned}$$

Especially higher moments increase the nonlinearity of the resulting surrogate models. But since the surrogate is deterministic every optimization method in the field of nonlinear programming (including metaheuristics) is in principle applicable to stochastic programming problems [13, p. 618; 12; p. 978].

Another difficulty is given by the fact that the evaluation of the objective function(s) and constraints may become time-consuming as an analytic determination of the moments is doomed to failure due to unsolvable integrals. Therefore every technique for saving function evaluations is highly welcome. This is the situation where the virtues of metaheuristics come into play.

2. Metaheuristics in Stochastic Programming

The deployment of metaheuristics in stochastic programming is wide-spread in case of combinatorial problems [4]. But there are comparatively few applications in the field of continuous and mixed-integer problems. This is somewhat surprising as there are a number of benefits when using these methods.

Consider the expectation model $E[\ f(x, \omega)\ ] \rightarrow \min!$ with x in R^n and random variable ω with probability density function $p_\omega(\cdot)$. Thus, the surrogate objective function $F: R^n \rightarrow R$ is

$$F(x) = E[\ f(x, \omega)\ ] = \int f(x, \omega)\, p_\omega(\omega)\, d\omega\ .$$

In most practical situation the above integral cannot be solved analytically and we must seek remedy in numerical integration. If the support of the random variable is not bounded the series has infinitely many elements and we have to break the series after a finite number of elements. But even if the support is bounded the step size of the numerical integration cannot become infinitesimal small since the calculation must terminate. In any case the resulting value is an approximation. We can regard the difference between true value and approximation as a realization of additive noise in the objective function. Fortunately, we know that metaheuristics like evolutionary strategies (ES) can cope with noise [3,9]. This property can now be exploited to introduce a technique for saving evaluation time of the objective function: If the ES is far away from the optimum the noise may be large without preventing the ES from taking the right decisions for minimizing a noisy function. The closer the ES approaches the true optimum the smaller must be the noise in order minimize the true objective function effectively. In our situation we can control the strength of noise simply by adjusting the step size and range of numerical integration: In the beginning the step size may be large and we can skip the integration over the tails of the probability density function. The closer we get to the optimum the more accurate will be our numerical integration. This approach saves a lot of time in the evaluations of the objective function.

Another successful technique in the field of metaheuristics is the deployment of metamodells. A metamodel is a model of the true objective function, that can be evaluated much quicker than the true objective function. Actually, a metamodell is nothing more than a surrogate model. In the field of stochastic programming we use a metamodel as a surrogate model for the deterministic surrogate objective function of the stochastic problem. Again, the metamodel yields only an approximation which may be regarded as a realization of additive noise. Evidently, a perfect playground for evolutionary algorithms and other metaheuristics. We can even think of combining metamodels with the previous approach.

Special scenarios and hence formulations of stochastic problems can be addressed by special versions of metaheuristics. The surrogate model for the *nonlinear two-stage stochastic problem with recourse* is

$$\min\{ f(x) + E[g(x, \omega) : x \in X] \} \quad \text{with} \quad g(x, \omega) = \min\{ h(x, y, \omega) : y \in Y \} \text{ and random } \omega \in \Omega$$

where f: $X \rightarrow R$ is deterministic (but multimodal) with X as a subset of R^n whereas $g(\cdot)$ may be discrete or continuous. Here, nested or hybrid or memetic metaheuristics can be deployed: The outer method optimizes over X whereas the inner method optimizes of Y. As for the inner method, the approaches discussed above for circumventing the exact determination of the expectation can be used.

There is some variety in surrogate models for stochastic programming problems and it is quite likely that some of them have never been addressed by metaheuristics up to the present. But since metaheuristics can cope with nonlinearities, discontinuities, and noise they are well equipped methods for deployment in solving stochastic programming problems.

3. Future Work

In the field of stochastic programming the surrogate objective function is not necessarily a black box. Actually, we ought to know which surrogate approach has been chosen. Moreover, we ought to know the distributions of the random variables. Therefore, it is reasonable to identify which (combinations of) metaheuristics are best suited for which surrogate approach and involved probability distributions. A theoretically supported recommendation would be highly appreciated in the practitioners' world.

There exists a lot of theoretical work on evolutionary strategies on noisy objective functions. These results can be combined with results from numerical integration theory to devise effective mechanisms for controlling the 'noise' in the objective function caused by the approximation of the integral(s).

Getting the 'noise' of the metamodels under control theoretically is a more ambitious task that requires also results from approximation theory in general. This topic will be postponed until first results for the previous scientific problem are available.

References

[1] E. Alba (ed.): *Parallel Metaheuristics*. A New Class of Algorithms. Wiley 2005.

[2] D. V. Arnold and H.-G. Beyer: A comparison of evolution strategies with other direct search methods in the presence of noise. *Computational Optimization and Applications, 24(1):135-159*, 2003.

[3] H.-G. Beyer and B. Sendhoff: Robust optimization—a comprehensive survey. *Computer Methods in Applied Mechanics and Engineering 196:3190-3218*, 2007.

[4] L. Bianchi, M. Dorigo, L.M. Gambardella and W.J. Gutjahr: A survey on metaheuristics for stochastic combinatorial optimization. *Natural Computation*, DOI 10.1007/s11047-008-9098-4, online: Sep. 2008.

[5] J. Branke: *Evolutionary Optimization in Dynamic Environments*. Kluwer, 2001.

[6] E. K. Burke and G. Kendall (ed.): *Search Methodologies*. Springer: New York 2005.

[7] F. W. Glover and G. A. Kochenberger (Hrsg.): *Handbook of Metaheuristics*. Springer Netherlands 2003.

[8] H. Hoos and Th. Stützle: *Stochastic Local Search: Foundations and Applications*. Morgan Kaufmann 2004.

[9] Y. Jin and J. Branke: Evolutionary optimization in uncertain environments—a survey. *IEEE Transactions in Evolutionary Computation 9(3):303-317*, 2005.

[10] T.G. Kolda, R.M. Lewis, and V. Torczon: Optimization by direct search: new perspectives on some classical and modern methods. *SIAM Review 45(3):385-482*, 2003.

[11] A. Ruszczynski and A. Shapiro (Hrsg.): *Stochastic Programming. Handbooks in Operations Research and Management Series*, Vol. 10. North Holland, Amsterdam 2003.
[12] N. V. Sahinidis: Optimization under uncertainty: state-of-the-art and opportunities. *Computers & Chemical Engineering 28:971-983*, 2004.
[13] R. Wets: Stochastic Programming, pp. 573-629 in Nemhauser et al. (ed.): *Optimization. Handbook in Operations Research and Management Science,* Vol. 1. North-Holland: Amsterdam 1989.