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Features of Easy and Hard Instances for Approximation Algorithms and the Traveling Salesperson Problem

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Features of Easy and Hard Instances for Approximation Algorithms and the Traveling Salesperson Problem

or

"What makes TSP instances difficult?"

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Summary

- Understanding the behavior of TSP solvers is still difficult.

- Here:
  - Three algorithms
    - 2-Approximation
    - 3/2-Approximation (Christofides)
    - Local search 2-opt*
  - Feature-based comparison
    - what makes instances hard/easy
  - Cross-comparison

School of Computer Science

Motivation, and the Travelling Salesperson Problem (TSP)

http://blogs.oc.edu/ee/images/uploads/ece/italyTour2_thumb.png

Motivation

- Understanding algorithm performance for hard optimization tasks is still difficult
- More precisely: given an instance I, it is often hard to predict the performance of an algorithm A, without running A on I.
- Classical approach (worst/average case) hardly captures real-world performance
- Hyper heuristics (optimization domain, machine learning) focus on finding the conditions that determine the algorithm performance in advance.
- Understanding important for (automated) algorithm selection.

Motivation

- Smith-Miles, Lopez [8] classify directions
  - Automatic algorithm selection is based on (learned knowledge from) previous algorithm performance
  - Analyze algorithms and problems theoretically/experimentally, to understand the reasons, to influence future algorithm design for more complex problems
- Here: we do both
  - We generate hard/easy instances, and characterize them
  - Insights can be used for performance prediction to support algorithm selection.

Travelling Salesperson Problem (TSP)

- Famous combinatorial NP-hard problem
- Given a set of *n* cities {1,…,n}, and a distance matrix $d=(d_{ij})$, $1 \leq i,j \leq n$, the task is to compute a tour of minimal length, which visits each cities once, and returns to the origin.
- Euclidian TSP (cities in the plane, Euclidian distances) is still NP-hard, and a special case of the Metric TSP (distances fulfill triangle inequalities, different approximation algorithms are known)

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Instance Generation and Investigated Features

Instance Generation

Approach by Mersmann et al. [6]

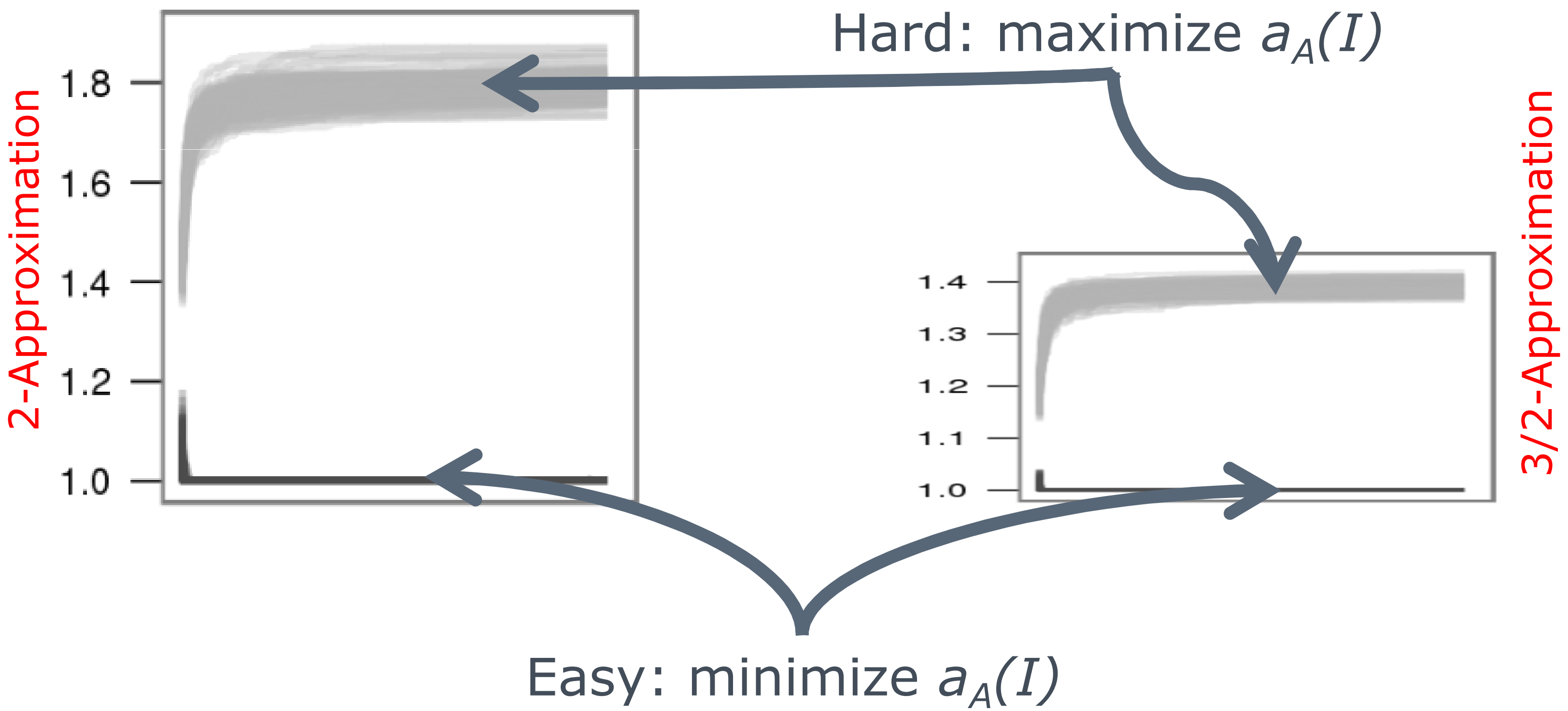
- Evolve hard/easy instances with an evolutionary algorithms
- Difficulty assessment (other measures possible)
  - $a_A(I)$ approximation ratio of algorithm A on instance I
  - $a_A(I)=A(I)/OPT(I)$

Note:

- an algorithm is a r-approximation algorithm,
  if $a_A(I) \leq r$ holds for all instances.
- $OPT(I)$ computed by CONCORDE [1]

Instance Generation

Use non-determinism: repeat runs for collections



Investigated Features

Mersmann et al. [6] (total: 46 features)

- Distance features: the cost distribution …
- Cluster features: number of cluster, mean distance to centroid, …
- Nearest Neighbour distance features: minimum, maximum, mean, standard deviation, …
- Centroid features: coordinates, distance to other nodes, …
- MST features: min/max/stdev/… of depth/distance values,…
- Angle features (between node and its two NN): min/max/…
- Convex Hull features: area, number of nodes defining CH, …

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Analysis: 2-Approximation

3/2-Approximation

Algorithm 1: 2-Approximation algorithm

input : Graph G

output: Hamiltonian cycle S

Subgraph connecting all vertices, with minimum weight

Build a minimum spanning tree MST T of G;

Duplicate all edges, forming the graph U;

Create an Euler cycle S in U;

Make S Hamiltonian by skipping already seen nodes;

return S;

improvement

Algorithm 2: Christofides 3/2-approximation algorithm

Set of pair-wise non-adjacent edges

⋮

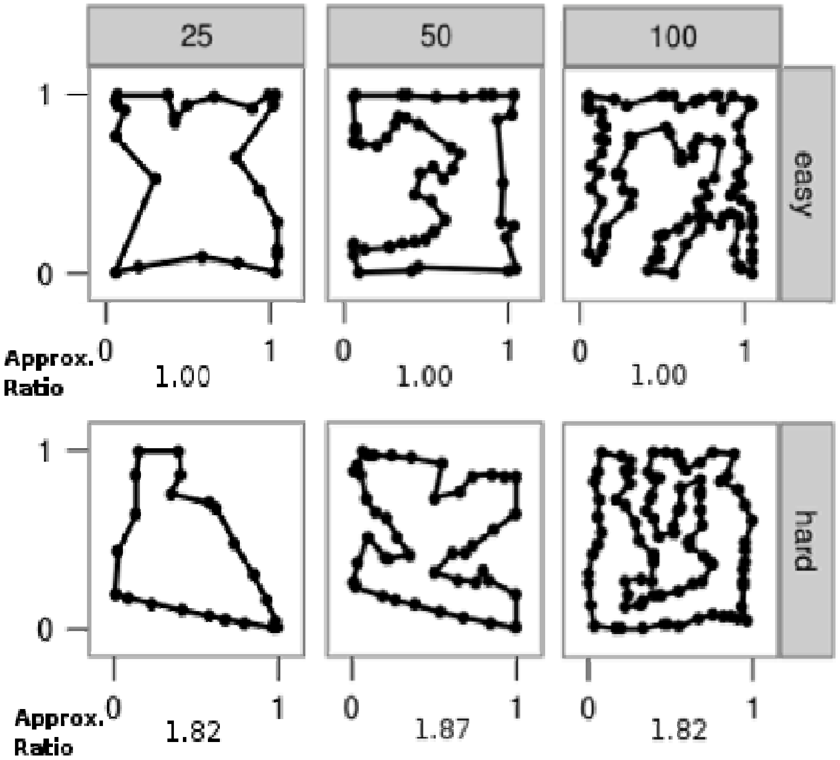
Find a minimum-weight perfect matching M on the set of nodes having an odd degree;

Combine the edges of M and T to form the graph U;

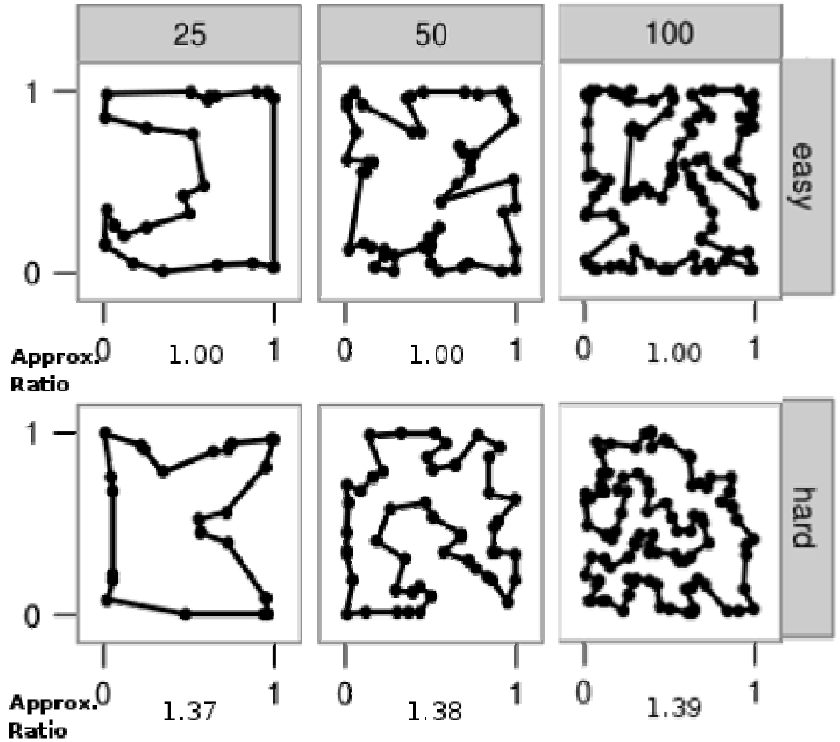
⋮

Step 1: Eye-Balling

2-Approximation

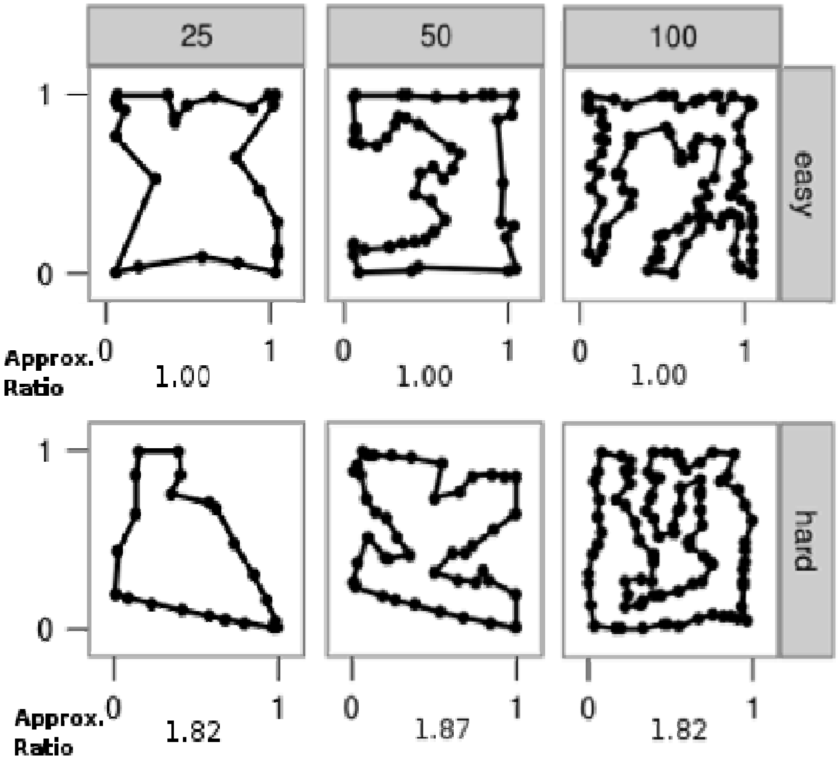


3/2-Approximation



Step 2: Analysis

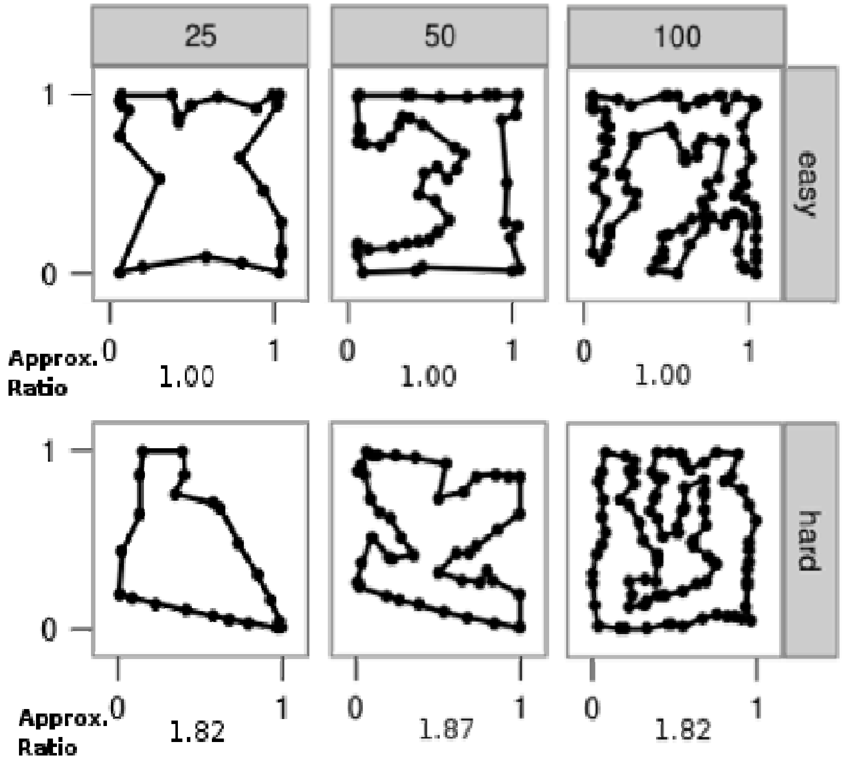
2-Approximation



- Distances of cities (optimal tour) are more uniform in the hard instances
- Standard deviations of the distances (optimal tour) of the easy instances are roughly twice as high than for the hard instances when considering small instance sizes. (decreases with *n*)
- Easy instances: small clusters (more uniform distribution in the hard instances)

Step 2: Analysis

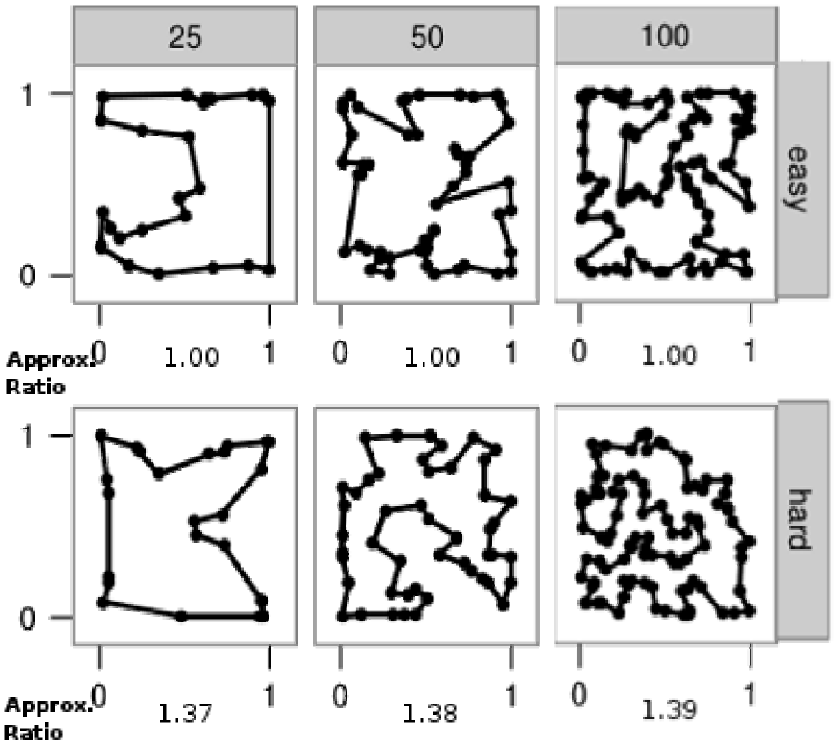
2-Approximation



- Easy instances: smaller angles (optimal tours)
- Mean angle values for easy/hard instances slightly decrease with the instance size.
- Instance shapes for small instances structurally differ from the respective shapes of larger instances. Consequently, the area covered by the convex hull is higher for larger instances.

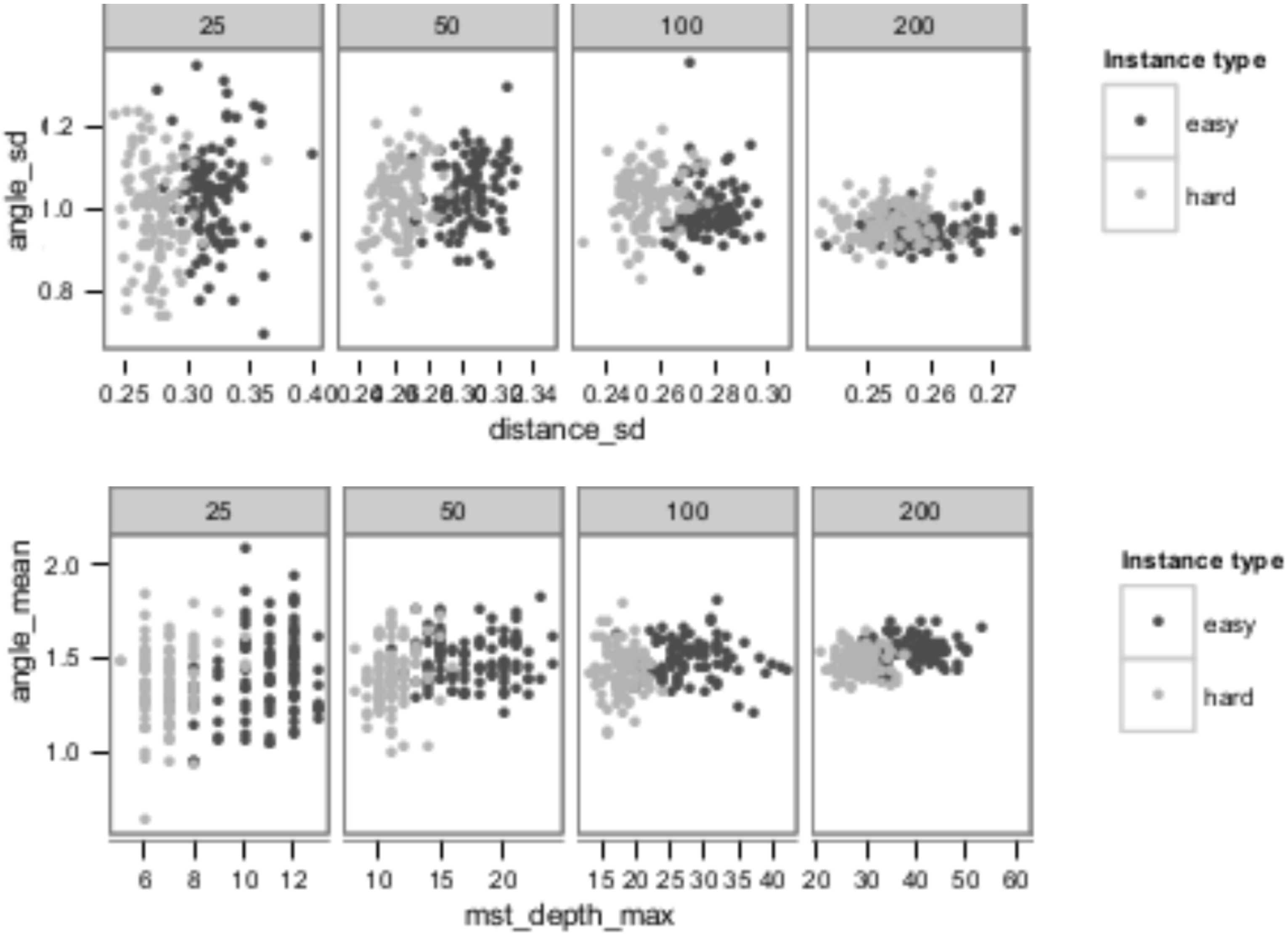
Step 2: Analysis

3/2-Approximation



- Visually, easy and hard instances do not differ significantly.
- Easy instances: considerably higher standard deviations of the distances (optimal tour).
- Mean angles: higher for (smaller) easy instances, lower for (larger) easy instances.

Exemplarily: Differentiating the generated Christofides Instances



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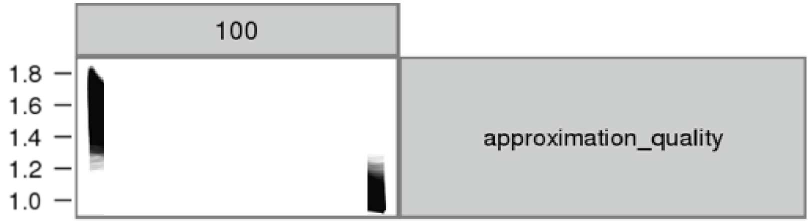
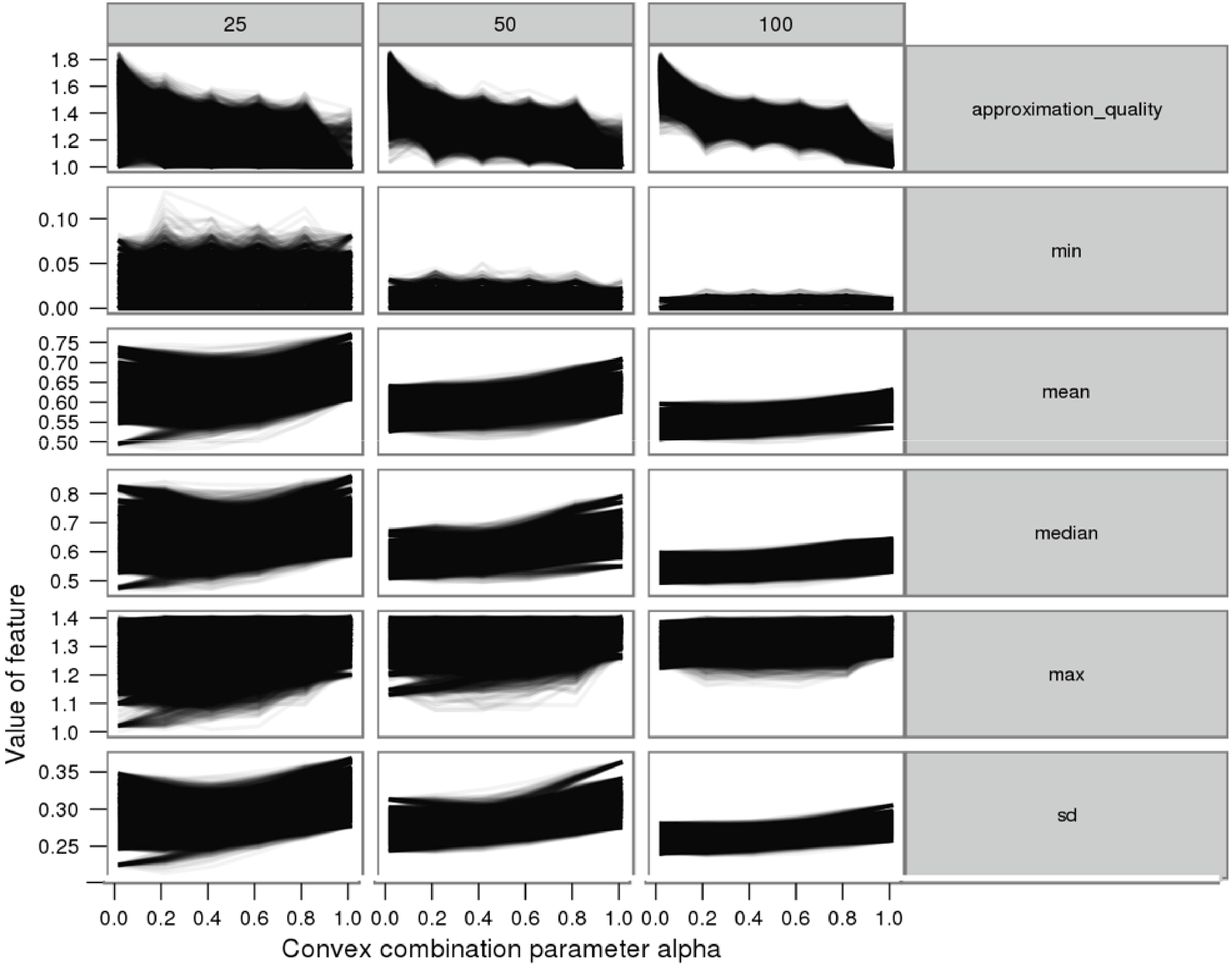
Morphed (Intermediate) Instances

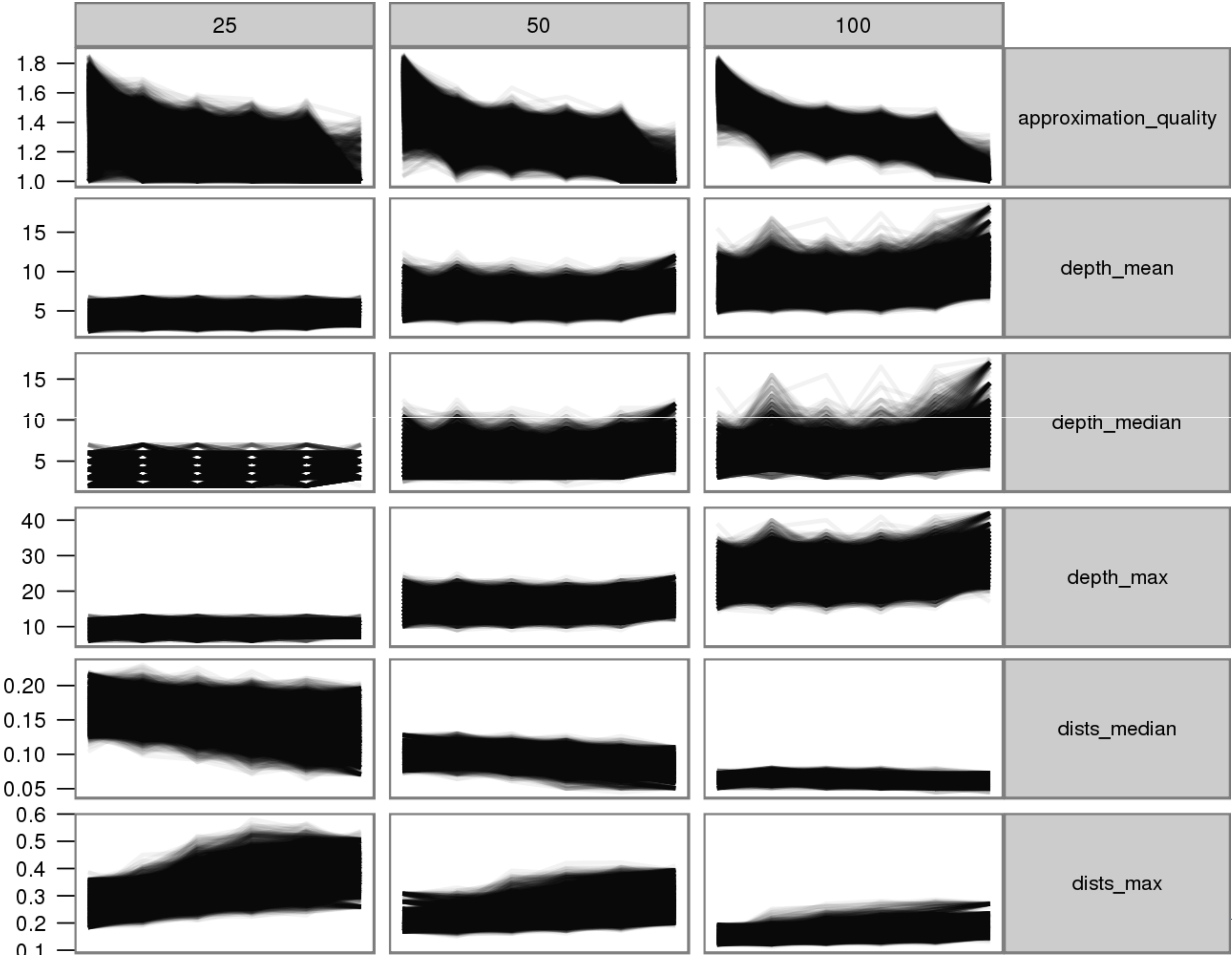
2-Approximation: distance features



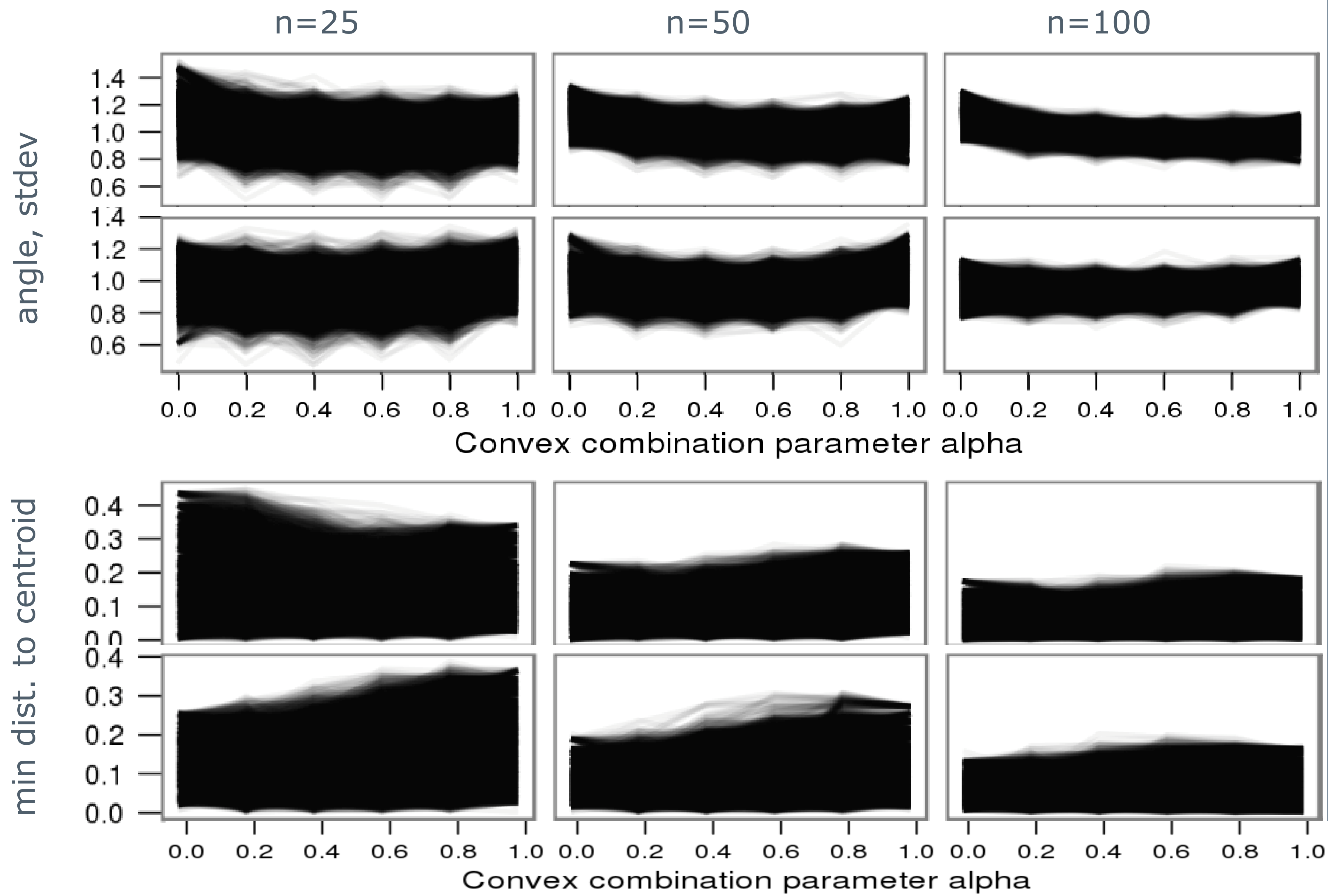
2-Approximation: distance features



2-Approximation: MST features



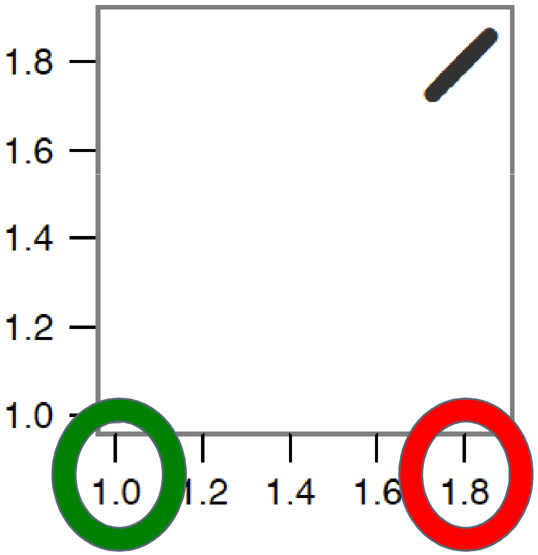
2-Approximation vs. 3/2-Approximation



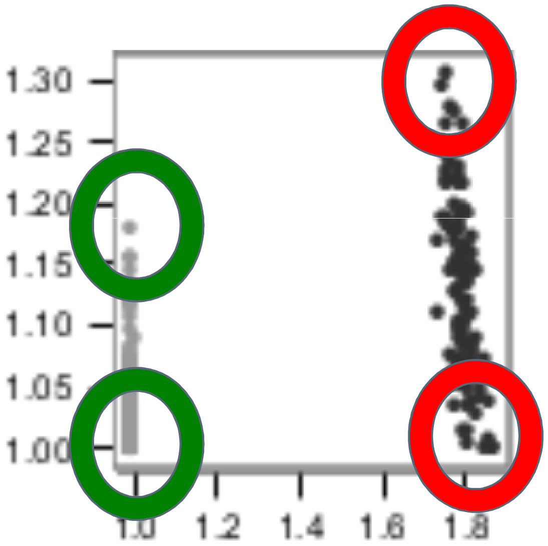
Cross-Comparison: 2-Approximation, 3/2-Approximation, 2-Opt

2-Approximation instances (n=25) on…

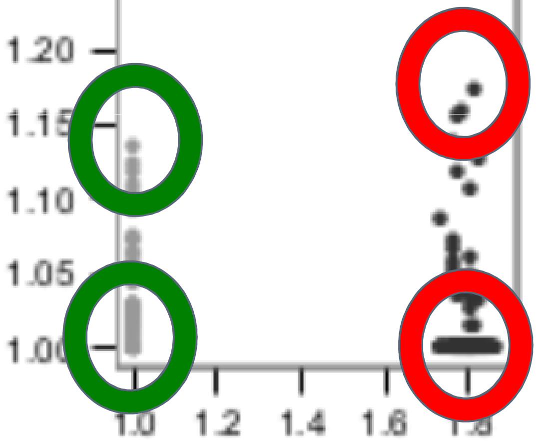
2-Approximation



3/2-Approximation

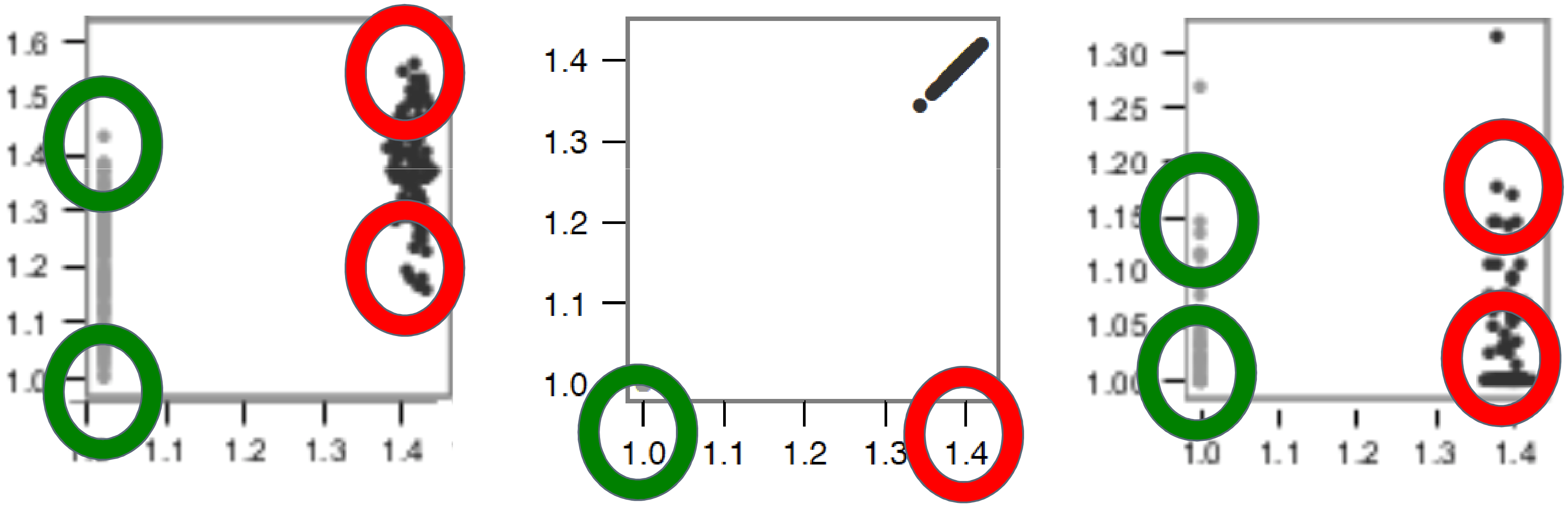


2-Opt



3/2-Approximation instances (n=25) on…

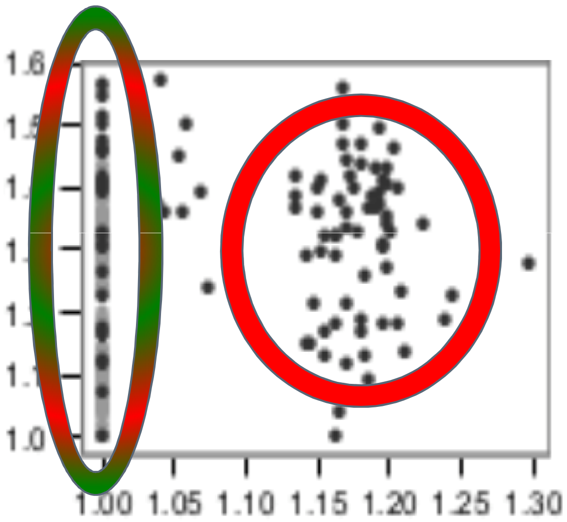
2-Approximation 3/2-Approximation 2-Opt



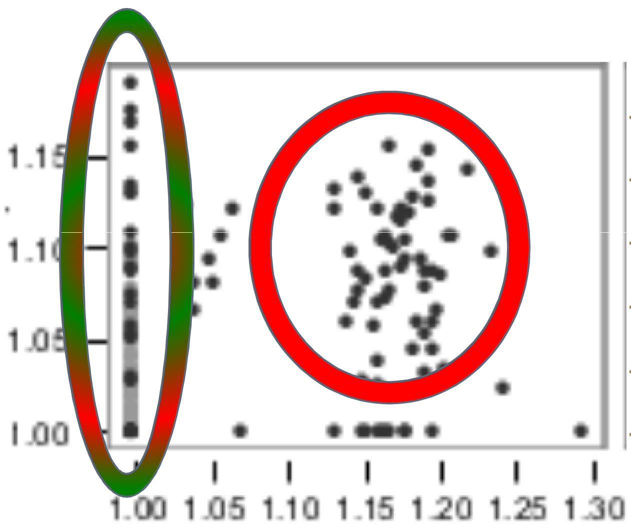
Note: 2-Opt “beats” 3/2-Approximation here (hard instances)

2-Opt instances (n=25) on...

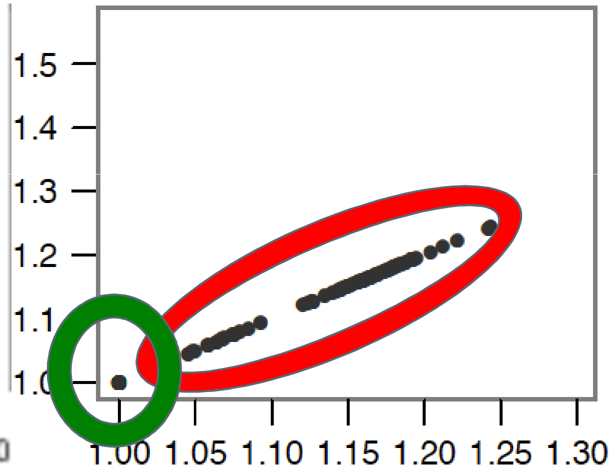
2-Approximation



3/2-Approximation



2-Opt



Note: 3/2-Approximation “beats” 2-Opt here (hard instances)

Thank you!