technische universität dortmund

Dept. of Computer Science
Chair of Algorithm Engineering

Tutorial for

# Introduction to Computational Intelligence in Winter 2015/16

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Lecture website: `https://tinyurl.com/CI-WS2015-16`

## Sheet 3, Block II

26 November 2015

**Due date: 09 December 2015, 2pm**
**Discussion: 10/11 December 2015**

**Exercise 3.1: Fuzzy Sets** (5 Points)

1. Consider the function $A(x) = \begin{cases} \frac{-(x-3)(x-7)}{3} & 3 \leq x \leq 7 \\ 0 & \text{otherwise} \end{cases}$. Could this function be used as a membership function? If not, modify the function so that it can.
2. Consider the functions $B(x) = \begin{cases} \frac{-(x-4)(x-8)}{4} & 4 \leq x \leq 8 \\ 0 & \text{otherwise} \end{cases}$ and $C(x) = \begin{cases} \frac{1}{4}x & 0 \leq x < 4 \\ 2 - \frac{1}{4}x & 4 \leq x \leq 8 \\ 0 & \text{otherwise} \end{cases}$.

   Let $D$ be the union of B and C, i.e. $D = B \cup C$, assuming standard operations for fuzzy sets.

   a) Calculate $D(3)$ and $D(7)$.

   b) Plot $D$ in the interval $[0, 8]$.

   c) Calculate $card(D)$.

**Solution (shortened version!)**

1. No, needs normalisation!
2. 

$$D(x) = \begin{cases} \frac{1}{4}x & 0 \leq x < 4 \\ 2 - \frac{1}{4}x & 4 \leq x < 5 \\ \frac{-(x-4)(x-8)}{4} & 5 \leq x \leq 8 \\ 0 & \text{otherwise} \end{cases}$$

$$card(D) = \int_{-\inf}^{\inf} D(x)dx = \int_0^4 \frac{1}{4}xdx + \int_4^5 (2 - \frac{1}{4}x)dx + \int_5^8 \frac{-(x-4)(x-8)}{4}dx = \frac{41}{8} = 5.125$$

**Exercise 3.2: t-norm** (4 Points)

Show that the Bounded difference is a t-norm.

**Solution (shortened version!)**

$$bd(a, b) = \max\{0, a + b - 1\}$$

1. $bd(a,1) = \max\{0, a+1-1\} = \max\{0, a\} = a$

2. $bd(a,b) = \max\{0, a+b-1\} \overset{b\leq d}{\leq} \max\{0, a+d-1\} = bd(a,d)$

3. $bd(a,b) = \max\{0, a+b-1\} \overset{com.add.}{=} \max\{0, b+a-1\} = bd(b,a)$

4.

$$\begin{aligned} bd(a, bd(b,c)) &= bd(a, \max\{0, b+c-1\}) = \max\{0, a+\max\{0, b+c-1\}-1\} \\ &= \max\{0, a-1, a+b+c-2\} \overset{a\leq 1}{=} \max\{0, a+b+c-2\} \\ &\overset{c\leq 1}{=} \max\{0, c-1, a+b+c-2\} = \max\{0, c+\max\{0, a+b-1\}-1\} \\ &= bd(c, \max\{0, a+b-1\}) = bd(c(bd(a,b)) \end{aligned}$$

**Exercise 3.3: Fuzzy Complement** (3 Points)

Prove that a fuzzy complement obtained from any invertible increasing generator must be involutive, i.e., $\forall a \in [0,1] : c(c(a)) = a$.

**Solution**
Let $g : [0,1] \to \mathbb{R}$ be an increasing generator with $c(a) = g^{-1}(g(1) - g(a)), \forall a \in [0,1]$.
Repeated insertion yields:

$$\begin{aligned} c(c(a)) &= c\Big(g^{-1}\big(g(1) - g(a)\big)\Big) \\ &= g^{-1}\Big(g(1) - g\Big(g^{-1}\big(g(1) - g(a)\big)\Big)\Big) \\ &= g^{-1}\Big(g(1) - \big(g(1) - g(a)\big)\Big) \\ &= g^{-1}(g(a)) \\ &= a \end{aligned}$$

**Exercise 3.4: Dual Triples** (4 Points)

Prove that the following operator triples are dual triples.

(a) $t(a,b) = ab$,
$s(a,b) = a + b - ab$,
$c(a) = 1 - a$

(b) $t(a,b) = \max\{0, a+b-1\}$,
$s(a,b) = \min\{1, a+b\}$,
$c(a) = 1 - a$

**Solution**
We have to prove $c(t(a,b)) = s(c(a), c(b))$ and $c(s(a,b)) = t(c(a), c(b))$, so the De Morgan rules hold.
(a)

$$\begin{aligned} c(t(a,b)) &= 1 - ab = (1-a) + (1-b) - (1-a)(1-b) = s(c(a), c(b)) \\ c(s(a,b)) &= 1 - a - b + ab = (1-a)(1-b) = 1 - (a+b-ab) = t(c(a), c(b)) \end{aligned}$$

(b)

$$\begin{aligned} c(t(a,b)) &= c(\max\{0, a+b-1\}) = 1 - \max\{0, a+b-1\} = \min\{1, -a-b+2\} \\ s(c(a), c(b)) &= s(1-a, 1-b) = \min\{1, 1-a+1-b\} = \min\{1, -a-b+2\} \\ c(s(a,b)) &= c(\min\{1, a+b\}) = 1 - \min\{1, a+b\} = \max\{0, 1-a-b\} \\ t(c(a), c(b)) &= t(1-a, 1-b) = \max\{0, 1-a+1-b-1\} = \max\{0, 1-a-b\} \end{aligned}$$

**Exercise 3.5: Application of Fuzzy Sets** (4 Points)

Imagine you are a video game designer for a Jump'n'Run game. You decide to implement a slider that enables the player to change the difficulty of the game within the interval $[0, 1]$. To test this feature, you ask 100 people to rate different difficulty settings and decide whether they are easy, normal or hard. The data you collected is shown in table 1.

| difficulty | E | N | H |
|---|---|---|---|
| 0 | 100 | 0 | 0 |
| 0.1 | 80 | 20 | 0 |
| 0.2 | 60 | 30 | 10 |
| 0.3 | 40 | 40 | 20 |
| 0.4 | 20 | 50 | 30 |
| 0.5 | 0 | 60 | 40 |
| 0.6 | 0 | 50 | 50 |
| 0.7 | 0 | 40 | 60 |
| 0.8 | 0 | 30 | 70 |
| 0.9 | 0 | 20 | 80 |
| 1 | 0 | 10 | 90 |

Table 1: Survey results: Number of people that claimed the game was easy (E), normal (N) or hard (H) for 11 different difficulty settings

1. Using the data, approximate membership functions for fuzzy sets $E, N$ and $H$ expressing the degree of membership of a difficulty $x \in [0, 1]$ to the sets easy $E$, normal $N$ and hard $H$, respectively.
2. Assume you do another survey and you now ask people for their estimate of the membership certain difficulties to the fuzzy sets $E, N$ and $H$. For a specific difficulty $x \in [0, 1]$ according to the survey it holds that: $E(x) = 0.4$, $N(x) = 0.5$ and $H(x) = 0.2$. Propose a method to estimate the value of $x$ using the membership functions you defined earlier and explain your idea.