technische universität dortmund

Dept. of Computer Science
Chair of Algorithm Engineering

Tutorial for

Introduction to Computational Intelligence in Winter 2015/16

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Lecture website: `https://tinyurl.com/CI-WS2015-16`

Sheet 3, Block II

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Due date: 09 December 2015, 2pm
Discussion: 10/11 December 2015

Exercise 3.1: Fuzzy Sets (5 Points)

1. Consider the function $A(x) = \begin{cases} \frac{-(x-3)(x-7)}{3} & 3 \leq x \leq 7 \\ 0 & \text{otherwise} \end{cases}$. Could this function be used as a membership function? If not, modify the function so that it can.

2. Consider the functions $B(x) = \begin{cases} \frac{-(x-4)(x-8)}{4} & 4 \leq x \leq 8 \\ 0 & \text{otherwise} \end{cases}$ and $C(x) = \begin{cases} \frac{1}{4}x & 0 \leq x < 4 \\ 2 - \frac{1}{4}x & 4 \leq x \leq 8 \\ 0 & \text{otherwise} \end{cases}$.

   Let D be the union of B and C, i.e. $D = B \cup C$, assuming standard operations for fuzzy sets.

   a) Calculate $D(3)$ and $D(7)$.

   b) Plot D in the interval $[0, 8]$.

   c) Calculate $card(D)$.

Exercise 3.2: t-norm (4 Points)

Show that the Bounded difference is a t-norm.

Exercise 3.3: Fuzzy Complement (3 Points)

Prove that a fuzzy complement obtained from any invertible increasing generator must be involutive, i.e., $\forall a \in [0, 1] : c(c(a)) = a$.

Exercise 3.4: Dual Triples (4 Points)

Prove that the following operator triples are dual triples.

(a) $t(a, b) = ab$,
$s(a, b) = a + b - ab$,
$c(a) = 1 - a$

(b) $t(a, b) = \max\{0, a + b - 1\}$,
$s(a, b) = \min\{1, a + b\}$,
$c(a) = 1 - a$

Exercise 3.5: Application of Fuzzy Sets (4 Points)

Imagine you are a video game designer for a Jump'n'Run game. You decide to implement a slider that enables the player to change the difficulty of the game within the interval $[0, 1]$. To test this feature, you ask 100 people to rate different difficulty settings and decide whether they are easy, normal or hard. The data you collected is shown in table 1.

difficulty	E	N	H
0	100	0	0
0.1	80	20	0
0.2	60	30	10
0.3	40	40	20
0.4	20	50	30
0.5	0	60	40
0.6	0	50	50
0.7	0	40	60
0.8	0	30	70
0.9	0	20	80
1	0	10	90

Table 1: Survey results: Number of people that claimed the game was easy (E), normal (N) or hard (H) for 11 different difficulty settings

1. Using the data, approximate membership functions for fuzzy sets E, N and H expressing the degree of membership of a difficulty $x \in [0, 1]$ to the sets easy E, normal N and hard H, respectively.
2. Assume you do another survey and you now ask people for their estimate of the membership certain difficulties to the fuzzy sets E, N and H. For a specific difficulty $x \in [0, 1]$ according to the survey it holds that: $E(x) = 0.4$, $N(x) = 0.5$ and $H(x) = 0.2$. Propose a method to estimate the value of x using the membership functions you defined earlier and explain your idea.