Computational Intelligence

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- Approximate Reasoning

- Fuzzy Control

So far:

- p: IF X is A THEN Y is B

  $\rightarrow$ R(x, y) = Imp(A(x), B(y))

  rule as relation; fuzzy implication

- rule: IF X is A THEN Y is B
  fact: X is A'
  conclusion: Y is B'

  $\rightarrow$ B'(y) = $\sup_{x \in X}$ t(A'(x), R(x, y))

  composition rule of inference

Thus:

- B'(y) = $\sup_{x \in X}$ t(A'(x), Imp(A(x), B(y)))

given	: fuzzy rule
input	: fuzzy set A'
output	: fuzzy set B'

here:

$$A'(x) = \begin{cases} 1 & \text{for } x = x_0 \\ 0 & \text{otherwise} \end{cases}$$

crisp input!

$$B'(y) = \sup_{x \in X} t(\, A'(x),\, \text{Imp}(\, A(x),\, B(y)\,)\,)$$

$$= \begin{cases} \displaystyle\sup_{x \neq x_0} t(\, 0,\, \text{Imp}(\, A(x),\, B(y)\,)\,) & \text{for } x \neq x_0 \\[2mm] t(\, 1,\, \text{Imp}(\, A(x_0),\, B(y)\,)\,) & \text{for } x = x_0 \end{cases}$$

$$= \begin{cases} 0 & \text{for } x \neq x_0 \qquad \text{since } t(0, a) = 0 \\[2mm] \text{Imp}(\, A(x_0),\, B(y)\,) & \text{for } x = x_0 \qquad \text{since } t(a, 1) = a \end{cases}$$

Lemma:

a) t(a, 1) = a

b) t(a, b) ≤ min { a, b }

c) t(0, a) = 0

Proof:

by a)

ad a) Identical to axiom 1 of t-norms.

ad b) From monotonicity (axiom 2) follows for b ≤ 1, that t(a, b) ≤ t(a, 1) = a.
      Commutativity (axiom 3) and monotonicity lead in case of a ≤ 1 to
      t(a, b) = t(b, a) ≤ t(b, 1) = b. Thus, t(a, b) is less than or
      equal to a as well as b, which in turn implies t(a, b) ≤ min{ a, b }.

ad c) From b) follows 0 ≤ t(0, a) ≤ min { 0, a } = 0 and therefore t(0, a) = 0. ∎

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Multiple rules:

IF X is A_1, THEN Y is B_1 $\rightarrow R_1(x, y) = Imp_1(A_1(x), B_1(y))$

IF X is A_2, THEN Y is B_2 $\rightarrow R_2(x, y) = Imp_2(A_2(x), B_2(y))$

IF X is A_3, THEN Y is B_3 $\rightarrow R_3(x, y) = Imp_3(A_3(x), B_3(y))$

…   …

IF X is A_n, THEN Y is B_n $\rightarrow R_n(x, y) = Imp_n(A_n(x), B_n(y))$

X is A'
_____

Y is B'

Multiple rules for <u>crisp input</u>: x_0 is given

$B_1'(y) = Imp_1(A_1(x_0), B_1(y))$

…

$B_n'(y) = Imp_n(A_n(x_0), B_n(y))$

                                     aggregation of rules or

                              local inferences necessary!

aggregate! $\Rightarrow$ $B'(y) = $ `aggr`$\{ B_1'(y), …, B_n'(y) \}$,   where `aggr` $= \begin{cases} \text{min} \\ \text{max} \end{cases}$

FITA: "**F**irst **i**nference, **t**hen **a**ggregate!"

1. Each rule of the form IF X is A_k THEN Y is B_k must be transformed by an appropriate fuzzy implication $Imp_k(\cdot, \cdot)$ to a relation R_k :
    $R_k(x, y) = Imp_k(A_k(x), B_k(y))$.

2. Determine $B_k'(y) = R_k(x, y) \circ A'(x)$ for all $k = 1, \ldots, n$ (local inference).

3. Aggregate to $B'(y) = \beta(B_1'(y), \ldots, B_n'(y))$.

FATI: "**F**irst **a**ggregate, **t**hen **i**nference!"

1. Each rule of the form IF X ist A_k THEN Y ist B_k must be transformed by an appropriate fuzzy implication $Imp_k(\cdot, \cdot)$ to a relation R_k :
    $R_k(x, y) = Imp_k(A_k(x), B_k(y))$.

2. Aggregate $R_1, \ldots, R_n$ to a **superrelation** with aggregating function $\alpha(\cdot)$:
    $R(x, y) = \alpha(R_1(x, y), \ldots, R_n(x, y))$.

3. Determine $B'(y) = R(x, y) \circ A'(x)$ w.r.t. superrelation (inference).

1. Which principle is better? FITA or FATI?

2. Equivalence of FITA and FATI ?

FITA:　　　　$B'(y) = \beta(B_1'(y), \ldots, B_n'(y))$

　　　　　　　　　$= \beta(R_1(x, y) \circ A'(x), \ldots, R_n(x, y) \circ A'(x))$

FATI:　　　　$B'(y) = R(x, y) \circ A'(x)$

　　　　　　　　　$= \alpha(R_1(x, y), \ldots, R_n(x, y)) \circ A'(x)$

special case:

$$A'(x) = \begin{cases} 1 & \text{for } x = x_0 \\ 0 & \text{otherwise} \end{cases}$$

crisp input!

On the equivalence of FITA and FATI:

FITA:
$$B'(y) = \beta(B_1'(y), \ldots, B_n'(y))$$
$$= \beta(Imp_1(A_1(x_0), B_1(y)), \ldots, Imp_n(A_n(x_0), B_n(y)))$$

FATI:
$$B'(y) = R(x, y) \circ A'(x)$$
$$= \sup_{x \in X} t(A'(x), R(x, y)) \qquad \text{(from now: special case)}$$
$$= R(x_0, y)$$
$$= \alpha(Imp_1(A_1(x_0), B_1(y)), \ldots, Imp_n(A_n(x_0), B_n(y)))$$

evidently: sup-t-composition with arbitrary t-norm and $\alpha(\cdot) = \beta(\cdot)$

- **AND-connected premises**

IF $X_1 = A_{11}$ AND $X_2 = A_{12}$ AND … AND $X_m = A_{1m}$ THEN $Y = B_1$
…
IF $X_n = A_{n1}$ AND $X_2 = A_{n2}$ AND … AND $X_m = A_{nm}$ THEN $Y = B_n$

reduce to single premise for each rule k:

$A_k(x_1,…, x_m) = \min \{ A_{k1}(x_1), A_{k2}(x_2), …, A_{km}(x_m) \}$ or in general: t-norm

- **OR-connected premises**

IF $X_1 = A_{11}$ OR $X_2 = A_{12}$ OR … OR $X_m = A_{1m}$ THEN $Y = B_1$
…
IF $X_n = A_{n1}$ OR $X_2 = A_{n2}$ OR … OR $X_m = A_{nm}$ THEN $Y = B_n$

reduce to single premise for each rule k:

$A_k(x_1,…, x_m) = \max \{ A_{k1}(x_1), A_{k2}(x_2), …, A_{km}(x_m) \}$ or in general: s-norm

important:

● if rules of the form **IF X is A THEN Y is B** interpreted as <u>logical</u> implication

$\Rightarrow$ $R(x, y) = Imp(A(x), B(y))$ makes sense

● we obtain: $B'(y) = \sup_{x \in X} t(A'(x), R(x, y))$

$\Rightarrow$ the worse the match of premise $A'(x)$, the larger is the fuzzy set $B'(y)$

$\Rightarrow$ follows immediately from axiom 1: $a \leq b$ implies $Imp(a, z) \geq Imp(b, z)$

interpretation of output set B'(y):

● $B'(y)$ is the set of values that are still possible

● each rule leads to an additional restriction of the values that are still possible

$\Rightarrow$ resulting fuzzy sets $B'_k(y)$ obtained from single rules must be mutually <u>intersected</u>!

$\Rightarrow$ aggregation via $\quad B'(y) = $ **min** $\{ B_1'(y), \ldots, B_n'(y) \}$

important:

● if rules of the form **IF *X* is A THEN *Y* is B** are <u>not</u> interpreted as <u>logical</u> implications, then the function Fct($\cdot$) in

$$R(x, y) = Fct(A(x), B(y))$$

can be chosen as required for desired interpretation.

● frequent choice (especially in fuzzy control):

   - R(x, y) = min { A(x), B(x) }   Mamdani – "implication"

   - R(x, y) = A(x) $\cdot$ B(x)   Larsen – "implication"

$\Rightarrow$ of course, they are no implications but special t-norms!

$\Rightarrow$ thus, if <u>relation R(x, y) is given</u>,
   then the *composition rule of inference*

$$B'(y) = A'(x) \circ R(x, y) = \sup_{x \in X} \min \{ A'(x), R(x, y) \}$$

still can lead to a conclusion via fuzzy logic.

example: [JM96, S. 244ff.]

industrial drill machine → control of cooling supply

modelling

linguistic variable   : **rotation speed**

linguistic terms   : *very low*, *low*, *medium*, *high*, *very high*

ground set   : $\mathcal{X}$ with $0 \leq x \leq 18000$ [rpm]
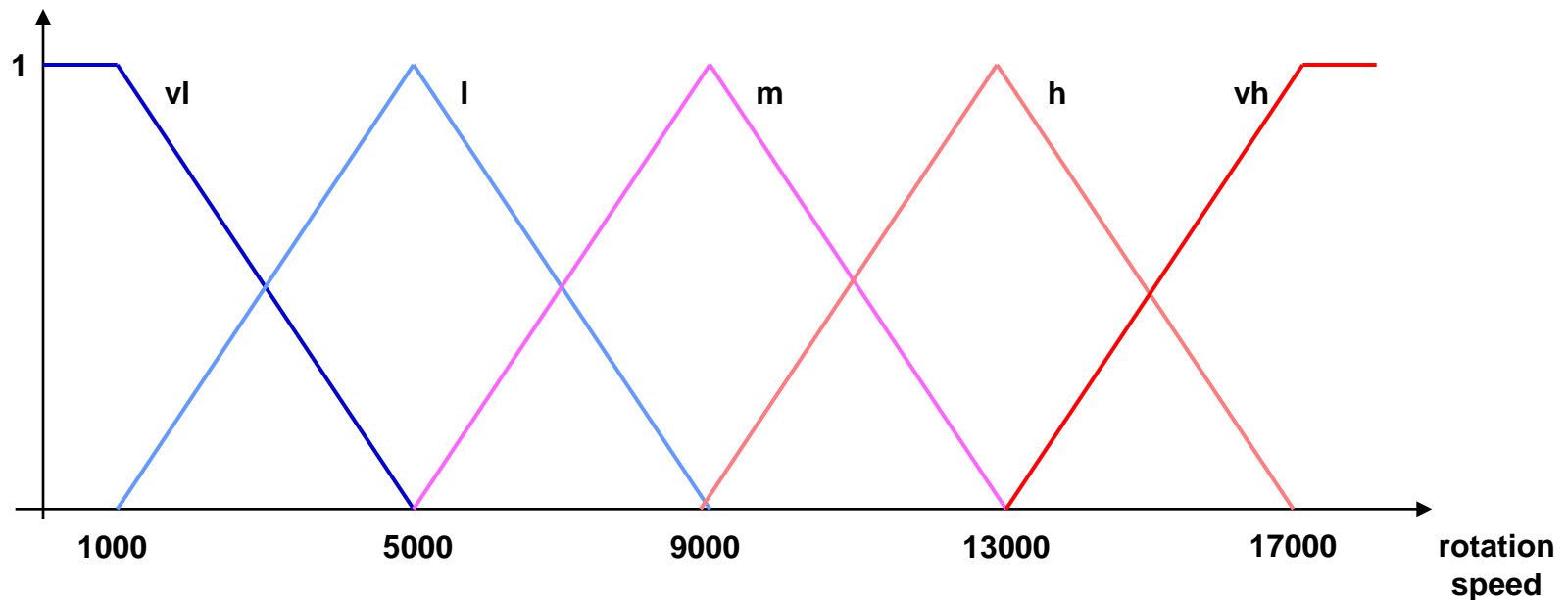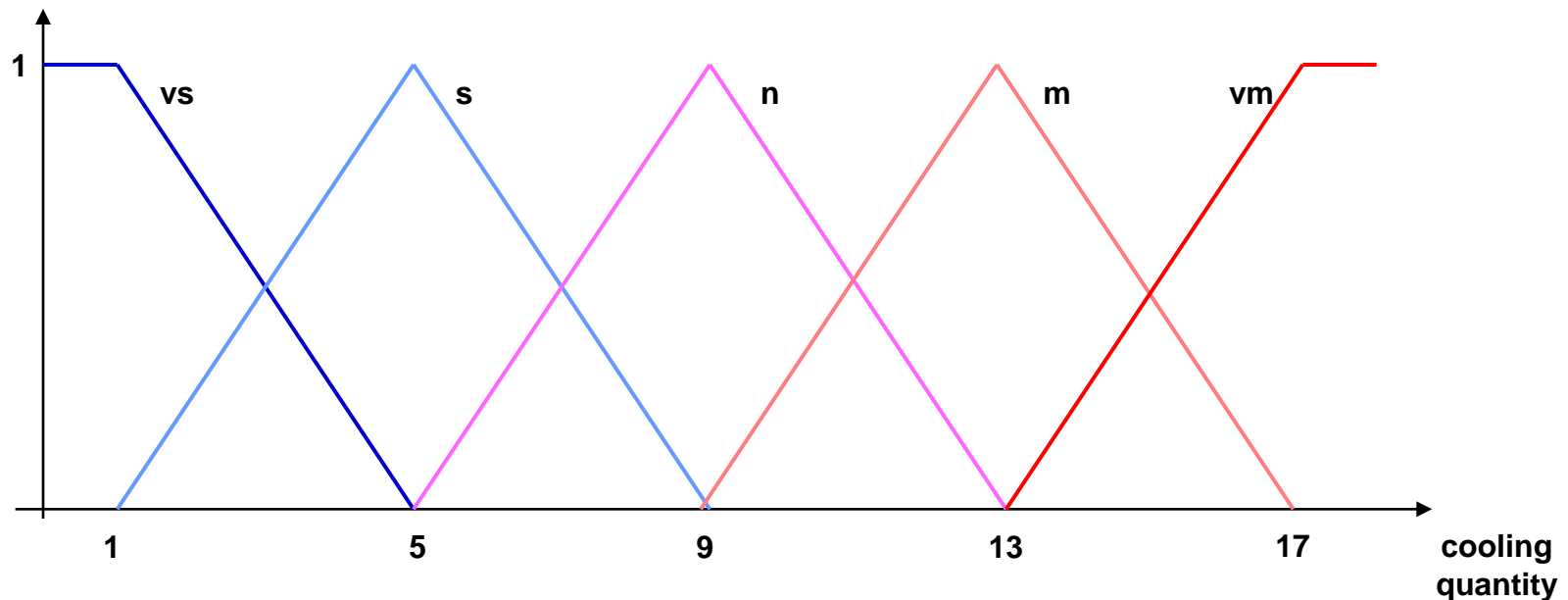
example: (continued)

industrial drill machine → control of cooling supply

modelling

linguistic variable   : **cooling quantity**

linguistic terms   : *very small*, *small*, *normal*, *much*, *very much*

ground set   : $\mathcal{Y}$ with $0 \leq y \leq 18$ [liter / time unit]

example: (continued)

industrial drill machine → control of cooling supply

<u>rule base</u>

IF rotation speed IS *very low* **THEN cooling quantity IS** *very small*		
low		*small*
medium		*normal*
high		*much*
very high		*very much*

sets S_{vl}, S_l, S_m, S_h, S_{vh}

"rotation <u>speed</u>"

sets C_{vs}, C_s, C_n, C_m, C_{vm}

"<u>cooling</u> quantity"

example: (continued)

industrial drill machine → control of cooling supply

1. **input**: crisp value $x_0 = 10000$ min^{-1} (no fuzzy set!)

    → **fuzzyfication** = determine membership for each fuzzy set over $\mathcal{X}$

    → yields S' = $(0, 0, \frac{3}{4}, \frac{1}{4}, 0)$ via $x \mapsto (\, S_{vl}(x_0),\, S_l(x_0),\, S_m(x_0),\, S_h(x_0),\, S_{vh}(x_0)\,)$

2. FITA: locale **inference** $\Rightarrow$ since $Imp(0,a) = 0$ we only need to consider:

    S_m: $C'_n(y) = Imp(\, \frac{3}{4},\, C_n(y)\,)$

    S_h: $C'_m(y) = Imp(\, \frac{1}{4},\, C_m(y)\,)$

3. **aggregation**:

    $C'(y) = $ `aggr` $\{\, C'_n(y),\, C'_m(y)\, \} = \boxed{max}\{\, Imp(\, \frac{3}{4},\, C_n(y)\,),\, Imp(\, \frac{1}{4},\, C_m(y)\,)\, \}$

?

?
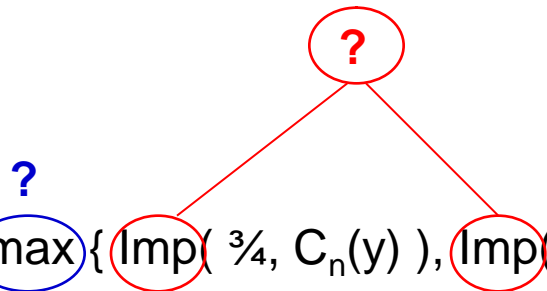
example: (continued)

industrial drill machine → control of cooling supply

$$C'(y) = \max \{ \text{Imp}(\tfrac{3}{4}, C_n(y)), \text{Imp}(\tfrac{1}{4}, C_m(y)) \}$$

in case of <u>control task</u> typically no logic-based interpretation:

→ max-aggregation and

→ relation $R(x,y)$ not interpreted as implication.

often: $R(x,y) = \min(a, b)$ „Mamdani controller"
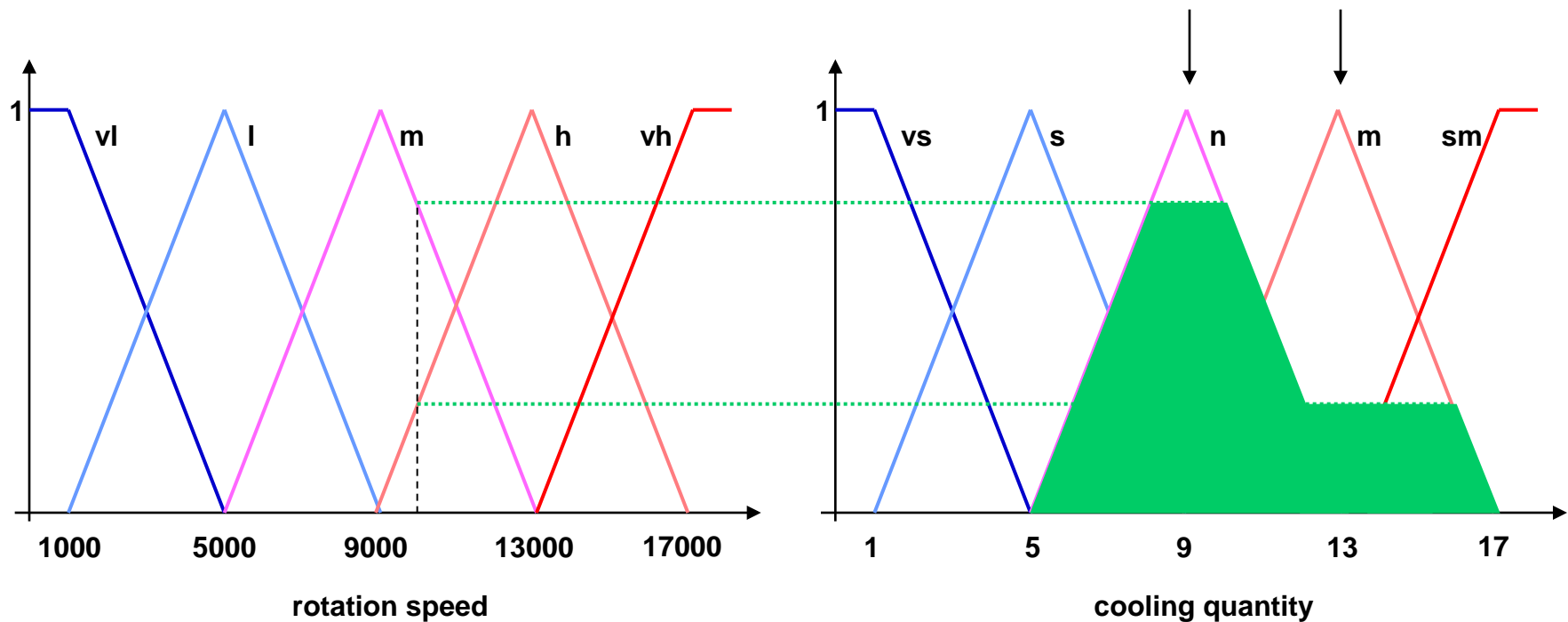
thus:

$$C'(y) = \max \{ \min \{ \tfrac{3}{4}, C_n(y) \}, \min \{ \tfrac{1}{4}, C_m(y) \} \}$$

→ graphical illustration

example: (continued)

industrial drill machine → control of cooling supply

$C'(y) = \max \{ \min \{ \frac{3}{4}, C_n(y) \}, \min \{ \frac{1}{4}, C_m(y) \} \}$, $x_0 = 10000$ [rpm]

rotation speed

cooling quantity

open and closed loop control:

affect the dynamical behavior of a system
in a desired manner

● open loop control

control is aware of reference values and has a model of the system
$\Rightarrow$ control values can be adjusted,
    such that process value of system is equal to reference value

problem: noise! $\Rightarrow$ deviation from reference value not detected

● closed loop control

now: detection of deviations from reference value possible
(by means of measurements / sensors)
and new control values can take into account the amount of deviation

open loop control

w
reference value

control value

control

u

system process

y
process value
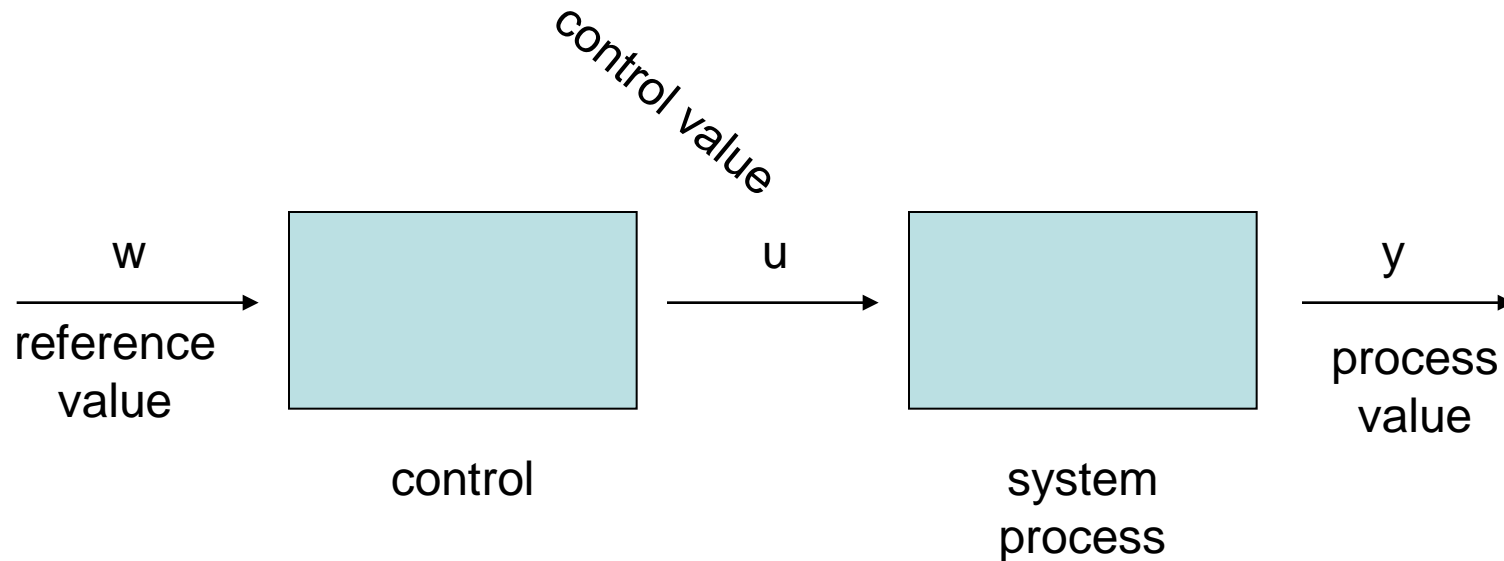
assumption: undisturbed operation $\Rightarrow$ process value = reference value

closed loop control
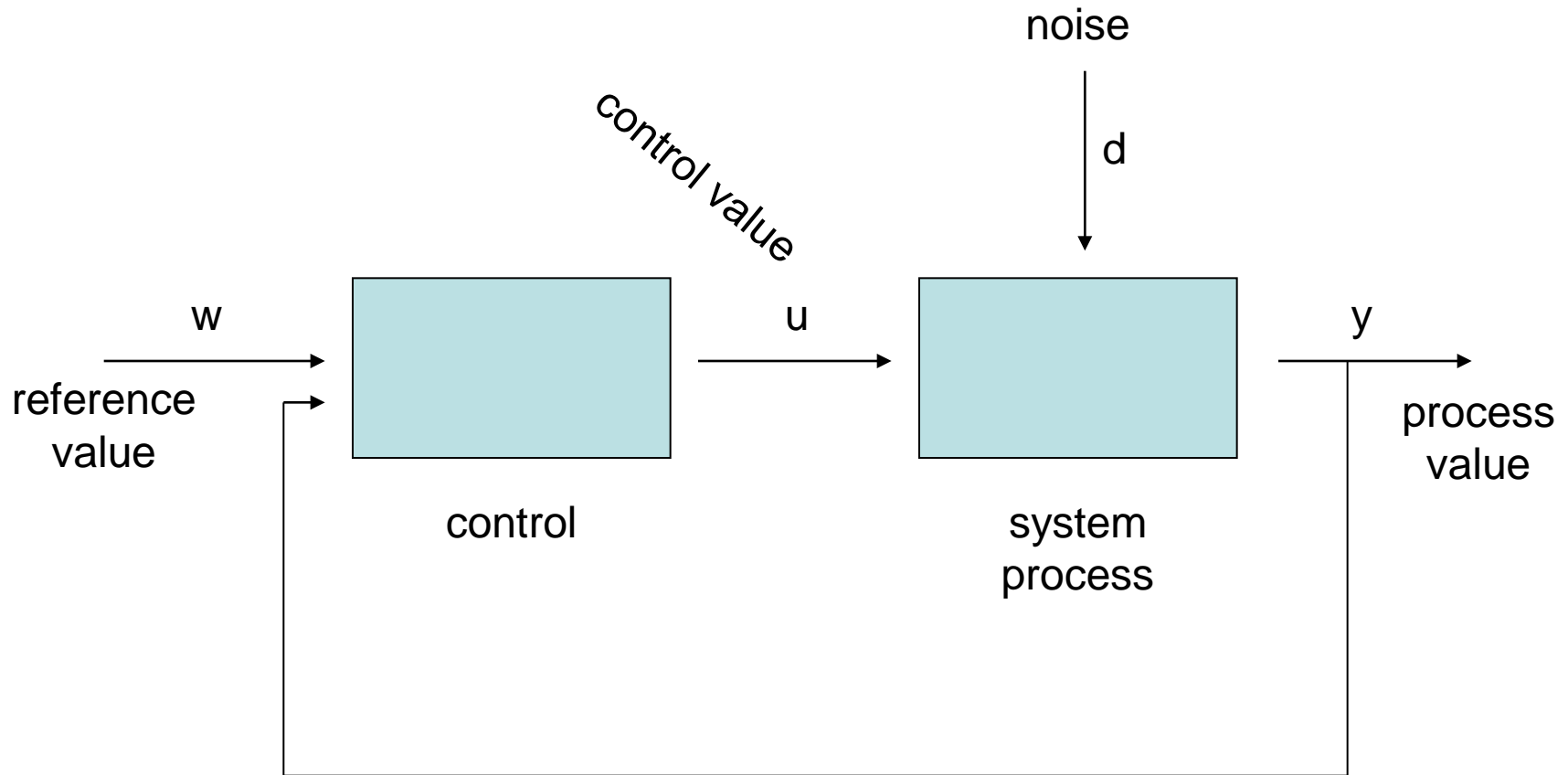
control deviation = reference value – process value

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required:

model of system / process

$\rightarrow$ as differential equations or difference equations (DEs)

$\rightarrow$ well developed theory available

so, why fuzzy control?

● there exists no process model in form of DEs etc.
  (operator/human being has realized control by hand)

● process with high-dimensional nonlinearities $\rightarrow$ no classic methods available

● control goals are vaguely formulated („soft" changing gears in cars)

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fuzzy description of control behavior

IF X is A_1, THEN Y is B_1
IF X is A_2, THEN Y is B_2
IF X is A_3, THEN Y is B_3

…

IF X is A_n, THEN Y is B_n
X is A'
_____

Y is B'

similar to approximative reasoning

but fact A' is not a fuzzy set but a crisp input

→ actually, it is the current process value

fuzzy controller executes inference step

→ yields fuzzy output set B'(y)

but crisp control value required for the process / system

→ defuzzification (= "condense" fuzzy set to crisp value)

defuzzification

Def: rule k active $\Leftrightarrow A_k(x_0) > 0$

● <u>maximum method</u>

- only active rule with largest activation level is taken into account

$\rightarrow$ suitable for pattern recognition / classification

$\rightarrow$ decision for a single alternative among finitely many alternatives

- selection independent from activation level of rule (0.05 vs. 0.95)

- if used for control: incontinuous curve of output values (leaps)

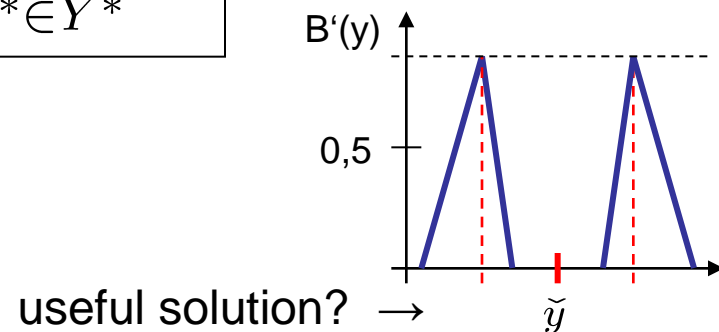$$\check{y} = \mathsf{argmax}\ B'(y)$$
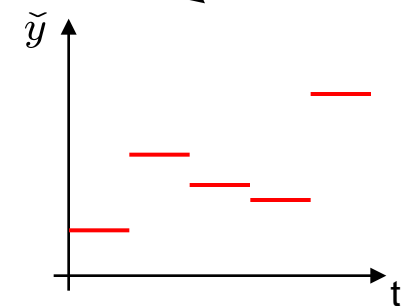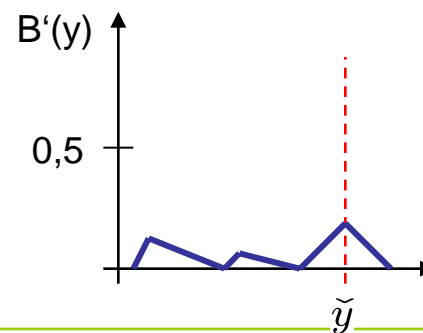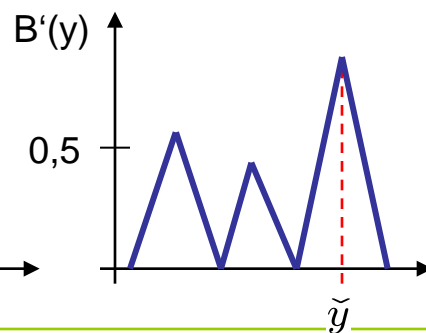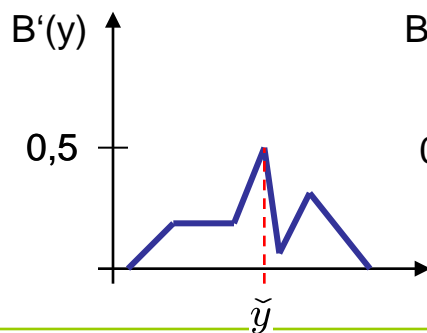
defuzzification

$$Y^* = \{\, y \in Y: B'(y) = hgt(B') \,\}$$

● <u>maximum mean value method</u>

    - all active rules with largest activation level are taken into account

        → interpolations possible, but need not be useful

        → obviously, only useful for neighboring rules with max. activation

    - selection independent from activation level of rule (0.05 vs. 0.95)

    - if used in control: incontinuous curve of output values (leaps)

$$\check{y} = \frac{1}{|Y^*|} \sum_{y^* \in Y^*} y^*$$

useful solution? →

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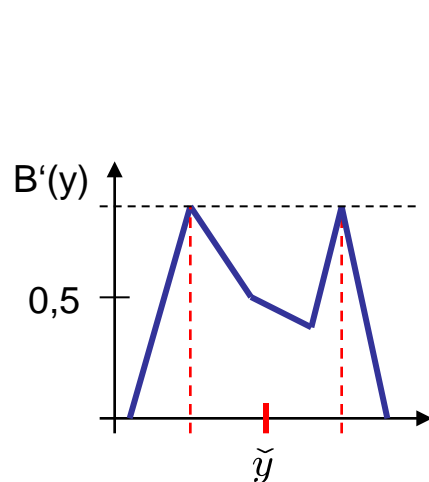
defuzzification

$$Y^* = \{\, y \in Y : B'(y) = hgt(B') \,\}$$

● <u>center-of-maxima method (COM)</u>

- only **extreme** active rules with largest activation level are taken into account

    → interpolations possible, but need not be useful

    → obviously, only useful for neighboring rules with max. activation level

- selection indepependent from activation level of rule (0.05 vs. 0.95)

- in case of control: incontinuous curve of output values (leaps)

$$\check{y} = \frac{\inf Y^* + \sup Y^*}{2}$$
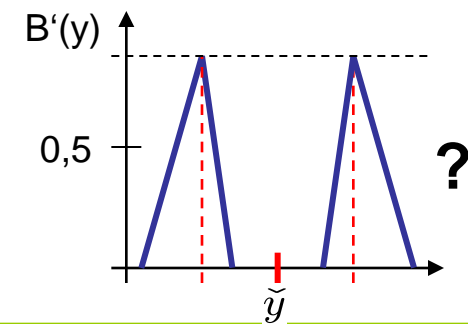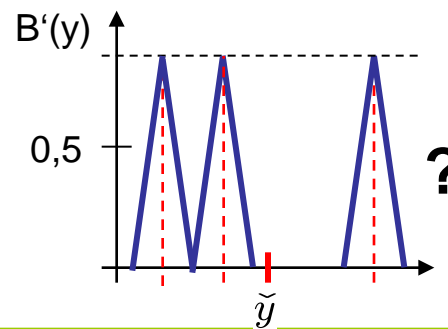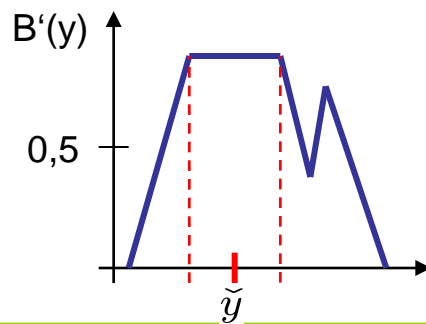
defuzzification

● Center of Gravity (COG)

- all active rules are taken into account

  → but numerically expensive …   …only valid for HW solution, today!

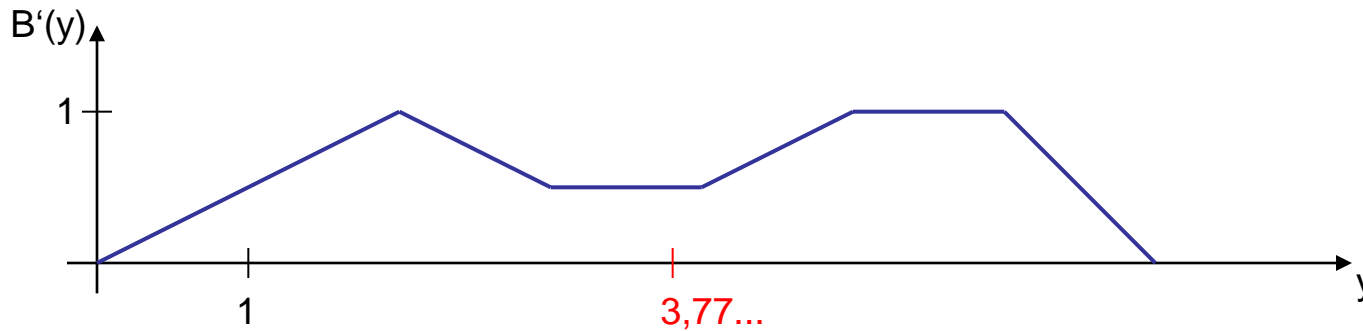 → borders cannot appear in output (∃ work-around)

- if only single active rule: independent from activation level

- continuous curve for output values

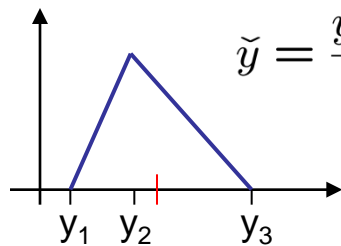$$\breve{y} = \frac{\int y \cdot B'(y)\, dy}{\int B'(y)\, dy}$$

Excursion: COG

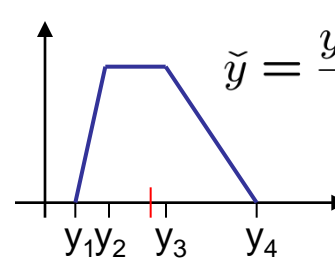$$\breve{y} = \frac{\int y \cdot B'(y)\, dy}{\int B'(y)\, dy}$$

pendant in
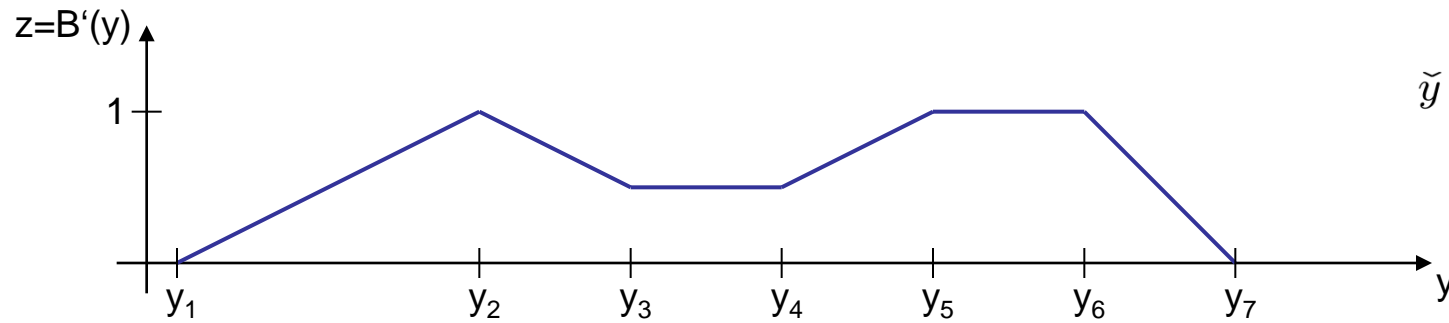probability theory:
expectation value

triangle:

$$\breve{y} = \frac{y_1 + y_2 + y_3}{3}$$

trapezoid:

$$\breve{y} = \frac{y_4^2 + y_3^2 - y_2^2 - y_1^2 + y_3 y_4 - y_1 y_2}{3\,(y_4 + y_3 - y_2 - y_1)}$$

$$\check{y} = \frac{\int y \cdot B'(y)\, dy}{\int B'(y)\, dy}$$

<u>assumption:</u> fuzzy membership functions piecewise linear

output set B'(y) represented by sequence of points (y_1, z_1), (y_2, z_2), …, (y_n, z_n)

$\Rightarrow$ area under B'(y) and weighted area can be determined additively piece by piece

$\Rightarrow$ linear equation $z = m\,y + b \Rightarrow$ insert (y_i, z_i) and (y_{i+1}, z_{i+1})

$\Rightarrow$ yields m and b for each of the n-1 linear sections

$$\Rightarrow F_i = \int_{y_i}^{y_{i+1}} (m\,y + b)\, dy = \frac{m}{2}(y_{i+1}^2 - y_i^2) + b(y_{i+1} - y_i)$$

$$\Rightarrow G_i = \int_{y_i}^{y_{i+1}} y\,(m\,y + b)\, dy = \frac{m}{3}(y_{i+1}^3 - y_i^3) + \frac{b}{2}(y_{i+1}^2 - y_i^2)$$

$$\check{y} = \frac{\sum\limits_i G_i}{\sum\limits_i F_i}$$

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Defuzzification

● Center of Area (COA)

　● developed as an approximation of COG

　● let $\hat{y}_k$ be the COGs of the output sets $B'_k(y)$:

$$\breve{y} = \frac{\sum_k A_k(x_0) \cdot \widehat{y}_k}{\sum_k A_k(x_0)}$$

how to:
assume that fuzzy sets $A_k(x)$ and $B_k(x)$ are triangles or trapezoids
let x_0 be the crisp input value
for each fuzzy rule "IF A_k is X THEN B_k is Y"
　determine $B'_k(y) = R(A_k(x_0), B_k(y))$, where $R(.,.)$ is the relation
　find $\hat{y}_k$ as center of gravity of $B'_k(y)$

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