technische universität dortmund

Computational Intelligence

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Plan for Today

- Application Fields of ANNs
  - Classification
  - Prediction
  - Function Approximation
- Radial Basis Function Nets (RBF Nets)
  - Model
  - Training
- Recurrent MLP
  - Elman Nets
  - Jordan Nets

Application Fields of ANNs

Classification

<u>given</u>: set of training patterns (input / output)

$\uparrow \tilde{x}_i$ $\quad \uparrow \tilde{y}_i$

output = label
(e.g. class A, class B, ...)

$$f(\underset{\substack{\downarrow \\ \text{input} \\ \text{(unknown)}}}{x};\ \overbrace{\underbrace{(\tilde{x}_1, \tilde{y}_1), \ldots, (\tilde{x}_m, \tilde{y}_m)}_{\substack{\text{training patterns} \\ \text{(known)}}}, \underbrace{w_1, \ldots, w_n}_{\substack{\text{weights} \\ \text{(learnt)}}}}^{\text{parameters}}) \rightarrow \underset{\substack{\downarrow \\ \text{output} \\ \text{(guessed)}}}{\hat{y}}$$

phase I:

train network

phase II:

apply network to unkown inputs for classification

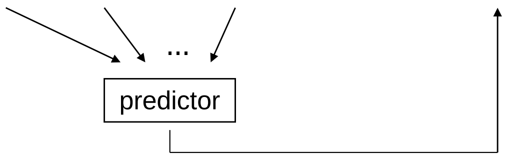
Application Fields of ANNs

Prediction of Time Series

time series $x_1, x_2, x_3, \ldots$ (e.g. temperatures, exchange rates, ...)

task: given a subset of historical data, predict the future

$$f(x_{t-k}, x_{t-k+1}, \ldots, x_t; w_1, \ldots, w_n) \rightarrow \hat{x}_{t+\tau}$$



training patterns:

historical data where true output is known;

error per pattern = $(\hat{x}_{t+\tau} - x_{t+\tau})^2$

phase I:

train network

phase II:

apply network to historical inputs for predicting <u>unkown</u> outputs

Prediction of Time Series: Example for Creating Training Data

given: time series 10.5, 3.4, 5.6, 2.4, 5.9, 8.4, 3.9, 4.4, 1.7

time window: k=3

(10.5, 3.4, 5.6) 2.4 first input / output pair

known input known output

further input / output pairs:

(3.4, 5.6, 2.4) 5.9

(5.6, 2.4, 5.9) 8.4

(2.4, 5.9, 8.4) 3.9

(5.9, 8.4, 3.9) 4.4

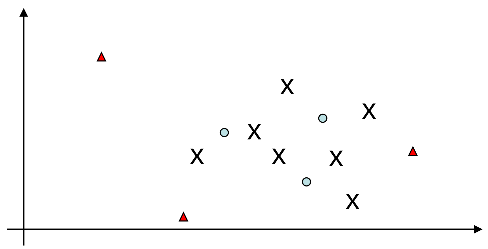
(8.4, 3.9, 4.4) 1.7

Function Approximation (the general case)

task: given training patterns (input / output), approximate unkown function

→ should give outputs close to true unkown function for arbitrary inputs

- values between training patterns are **interpolated**
- values outside convex hull of training patterns are **extrapolated**



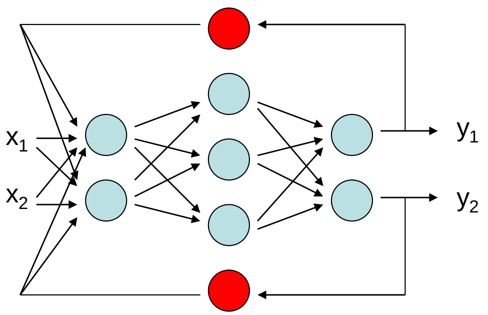
x : input training pattern

∘ : input pattern where output to be interpolated

▴ : input pattern where output to be extrapolated

Jordan nets (1986)

- **context neuron**:
  reads output from some neuron at step t and feeds value into net at step t+1



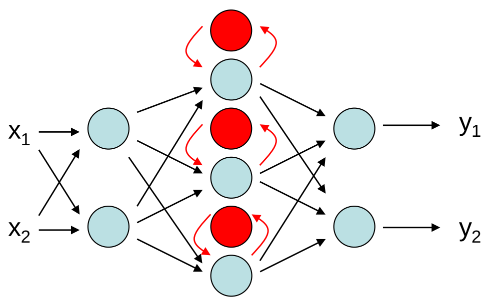
Jordan net =

MLP + context neuron for each output, context neurons fully connected to input layer

Elman nets (1990)

Elman net =

MLP + context neuron for each hidden layer neuron‘s output of MLP, context neurons fully connected to emitting MLP layer



Training?

$\Rightarrow$ unfolding in time ("loop unrolling")

- identical MLPs serially connected (finitely often)
- results in a large MLP with many hidden (inner) layers
- backpropagation may take a long time
- but reasonable if most recent past more important than layers far away

Why using backpropagation?

$\Rightarrow$ use *Evolutionary Algorithms* directly on recurrent MLP!

later!

Definition:
A function $\phi : \mathbb{R}^n \to \mathbb{R}$ is termed **radial basis function**
iff $\exists\, \varphi : \mathbb{R} \to \mathbb{R} : \forall\, x \in \mathbb{R}^n : \phi(x; c) = \varphi(\,\|\, x - c \,\|\,)$. □

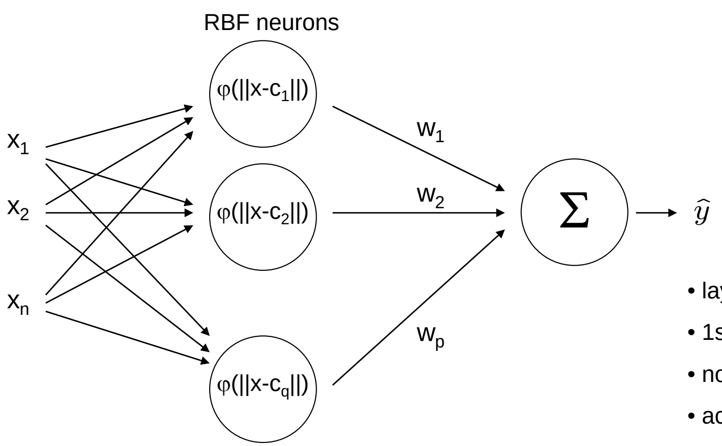
Definition:
RBF **local** iff
$\varphi(r) \to 0$ as $r \to \infty$ □

typically, $\|\, x \,\|$ denotes Euclidean norm of vector x

examples:

$\varphi(r) = \exp\left(-\dfrac{r^2}{\sigma^2}\right)$	Gaussian	unbounded	local
$\varphi(r) = \dfrac{3}{4}(1-r^2)\cdot 1_{\{r \le 1\}}$	Epanechnikov	bounded	local
$\varphi(r) = \dfrac{\pi}{4}\cos\left(\dfrac{\pi}{2}r\right)\cdot 1_{\{r \le 1\}}$	Cosine	bounded	local

Definition:
A function f: $\mathbb{R}^n \to \mathbb{R}$ is termed **radial basis function net (RBF net)**
iff $f(x) = w_1\, \varphi(\|\, x - c_1 \,\|\,) + w_2\, \varphi(\|\, x - c_2 \,\|\,) + \ldots + w_p\, \varphi(\|\, x - c_q \,\|\,)$ □



- layered net
- 1st layer fully connected
- no weights in 1st layer
- activation functions differ

given : N training patterns (x_i, y_i) and q RBF neurons

find : weights $w_1, \ldots, w_q$ with minimal error

solution:

we know that $f(x_i) = y_i$ for i = 1, ..., N and therefore we insist that

$$\underbrace{\sum_{k=1}^{q} w_k}_{\text{unknown}} \cdot \underbrace{\varphi(\|x_i - c_k\|)}_{p_{ik}\ \text{known value}} = \underbrace{y_i}_{\text{known value}}$$

$$\Rightarrow \quad \sum_{k=1}^{q} w_k \cdot p_{ik} = y_i$$

$\Rightarrow$ N linear equations with q unknowns

in matrix form: $P\,w = y$ with $P = (p_{ik})$ and $P: N \times q$, $y: N \times 1$, $w: q \times 1$,

case $N = q$: $w = P^{-1}\,y$ if P has full rank

case $N < q$: many solutions but of no practical relevance

case $N > q$: $w = P^{+}\,y$ where P^{+} is Moore-Penrose pseudo inverse

$P\,w = y$ | $\cdot$ P' from left hand side (P' is transpose of P)

$P'P\,w = P'\,y$ | $\cdot\ (P'P)^{-1}$ from left hand side

$\underbrace{(P'P)^{-1}\,P'P}_{\text{unit matrix}}\,w = \underbrace{(P'P)^{-1}\,P'}_{P^{+}}\,y$ | simplify

complexity (naive)

$w = (P'P)^{-1}\,P'\,y$

$$\underbrace{P'P: N^2 q \qquad \text{inversion: } q^3 \qquad P'y: qN \qquad \text{multiplication: } q^2}_{O(N^2\,q)}$$

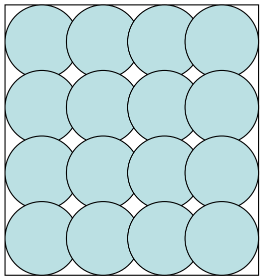
remark: if N large then inaccuracies for P'P likely

$\Rightarrow$ first analytic solution, then $\underbrace{\text{gradient descent}}_{\text{requires differentiable basis functions!}}$ starting from this solution

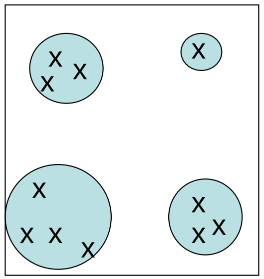
so far: tacitly assumed that RBF neurons are given

$\Rightarrow$ center c_k and radii σ considered given and known

how to choose c_k and σ ?



uniform covering



if training patterns inhomogenously distributed then first cluster analysis

choose center of basis function from each cluster, use cluster size for setting σ

advantages:

- additional training patterns → only local adjustment of weights
- optimal weights determinable in polynomial time
- regions not supported by RBF net can be identified by zero outputs

  (if output close to zero, verify that output of each basis function is close to zero)

disadvantages:

- number of neurons increases exponentially with input dimension
- unable to extrapolate (since there are no centers and RBFs are local)