technische universität dortmund

Computational Intelligence

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Plan for Today

- Application Fields of ANNs
  - Classification
  - Prediction
  - Function Approximation
- Radial Basis Function Nets (RBF Nets)
  - Model
  - Training
- Recurrent MLP
  - Elman Nets
  - Jordan Nets

Classification

given: set of training patterns (input / output)

↑ $\tilde{x}_i$ ↑ $\tilde{y}_i$

output = label
(e.g. class A, class B, ...)

$$f(x; \overbrace{\underbrace{(\tilde{x}_1, \tilde{y}_1), \ldots, (\tilde{x}_m, \tilde{y}_m)}_{\text{training patterns (known)}}, \underbrace{w_1, \ldots, w_n}_{\text{weights (learnt)}}}^{\text{parameters}}) \rightarrow \hat{y}$$

- x: input (unknown)
- training patterns (known)
- weights (learnt)
- $\hat{y}$: output (guessed)

phase I:

train network

phase II:

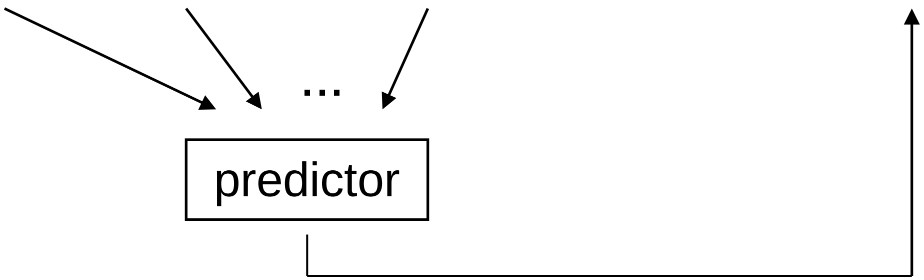
apply network to unkown inputs for classification

Prediction of Time Series

time series $x_1, x_2, x_3, \ldots$ (e.g. temperatures, exchange rates, ...)

task: given a subset of historical data, predict the future

$$f(x_{t-k}, x_{t-k+1}, \ldots, x_t; w_1, \ldots, w_n) \rightarrow \hat{x}_{t+\tau}$$



training patterns:

historical data where true output is known;

error per pattern = $(\hat{x}_{t+\tau} - x_{t+\tau})^2$

phase I:

train network

phase II:

apply network to historical inputs for predicting unkown outputs

Prediction of Time Series: Example for Creating Training Data

given: time series 10.5, 3.4, 5.6, 2.4, 5.9, 8.4, 3.9, 4.4, 1.7

time window: k=3

(10.5, 3.4, 5.6) 2.4 — first input / output pair

known input — known output

further input / output pairs:

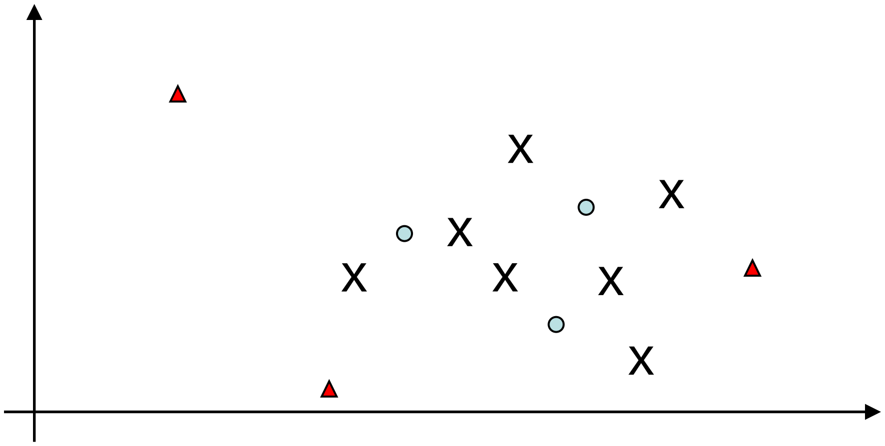
known input	known output
(3.4, 5.6, 2.4)	5.9
(5.6, 2.4, 5.9)	8.4
(2.4, 5.9, 8.4)	3.9
(5.9, 8.4, 3.9)	4.4
(8.4, 3.9, 4.4)	1.7

Function Approximation (the general case)

task: given training patterns (input / output), approximate unkown function

→ should give outputs close to true unkown function for arbitrary inputs

- values between training patterns are **interpolated**
- values outside convex hull of training patterns are **extrapolated**



x : input training pattern

○ : input pattern where output to be interpolated

▲ : input pattern where output to be extrapolated

Definition:

A function $\phi : \mathbb{R}^n \to \mathbb{R}$ is termed **radial basis function**

iff $\exists\, \varphi : \mathbb{R} \to \mathbb{R} : \forall\, x \in \mathbb{R}^n : \phi(x; c) = \varphi\,(\,\|\, x - c \,\|\,)$. □

Definition:

RBF **local** iff

$\varphi(r) \to 0$ as $r \to \infty$ □

typically, || x || denotes Euclidean norm of vector x

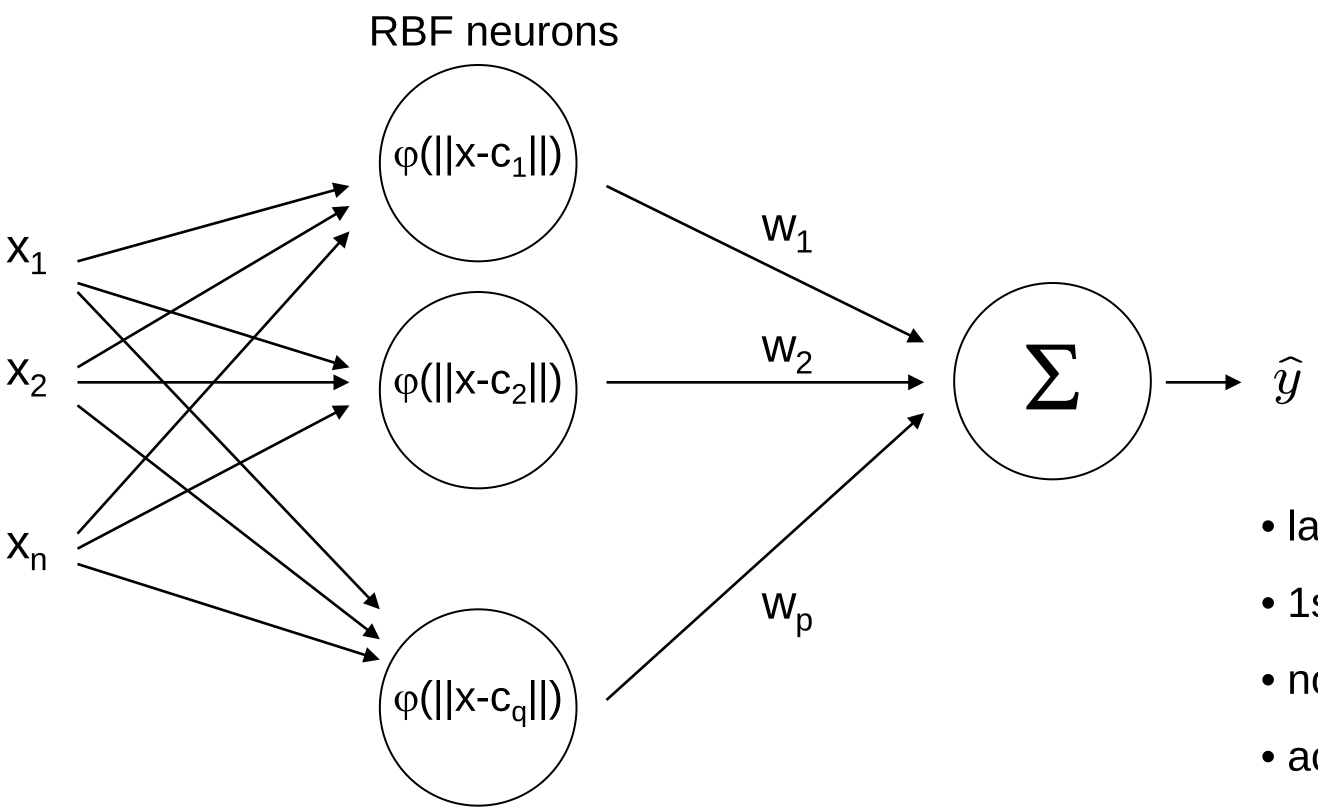
examples:

$\varphi(r) = \exp\left(-\frac{r^2}{\sigma^2}\right)$	Gaussian	unbounded	local
$\varphi(r) = \frac{3}{4}(1 - r^2) \cdot 1_{\{r \le 1\}}$	Epanechnikov	bounded	local
$\varphi(r) = \frac{\pi}{4} \cos\left(\frac{\pi}{2} r\right) \cdot 1_{\{r \le 1\}}$	Cosine	bounded	local

Definition:

A function f: $\mathbb{R}^n \rightarrow \mathbb{R}$ is termed **radial basis function net (RBF net)**

iff $f(x) = w_1\, \varphi(\| x - c_1 \|) + w_2\, \varphi(\| x - c_2 \|) + \ldots + w_p\, \varphi(\| x - c_q \|)$ □



- layered net
- 1st layer fully connected
- no weights in 1st layer
- activation functions differ

given : N training patterns (x_i, y_i) and q RBF neurons

find : weights $w_1, ..., w_q$ with minimal error

solution:

we know that $f(x_i) = y_i$ for i = 1, ..., N or equivalently

$$\sum_{k=1}^{q} w_k \cdot \underbrace{\varphi(\|x_i - c_k\|)}_{p_{ik}} = y_i$$

w_k → unknown; p_{ik} → known value; y_i → known value

$$\Rightarrow \quad \sum_{k=1}^{q} w_k \cdot p_{ik} = y_i$$

$\Rightarrow$ N linear equations with q unknowns

in matrix form: $P\,w = y$ with $P = (p_{ik})$ and $P: N \times q$, $y: N \times 1$, $w: q \times 1$,

case $N = q$: $w = P^{-1}\,y$ if P has full rank

case $N < q$: many solutions but of no practical relevance

case $N > q$: $w = P^{+}\,y$ where P^{+} is Moore-Penrose pseudo inverse

$P\,w = y$ | $\cdot$ P' from left hand side (P' is transpose of P)

$P'P\,w = P'\,y$ | $\cdot\ (P'P)^{-1}$ from left hand side

$$\underbrace{(P'P)^{-1}\,P'P}_{\text{unit matrix}}\,w = \underbrace{(P'P)^{-1}\,P'}_{P^{+}}\,y$$ | simplify

complexity (naive)

$w = (P'P)^{-1} P' y$

$P'P$: $N^2 q$ inversion: q^3 $P'y$: qN multiplication: q^2

$\underbrace{\qquad\qquad\qquad\qquad}_{O(N^2 q)}$

remark: if N large then inaccuracies for P'P likely

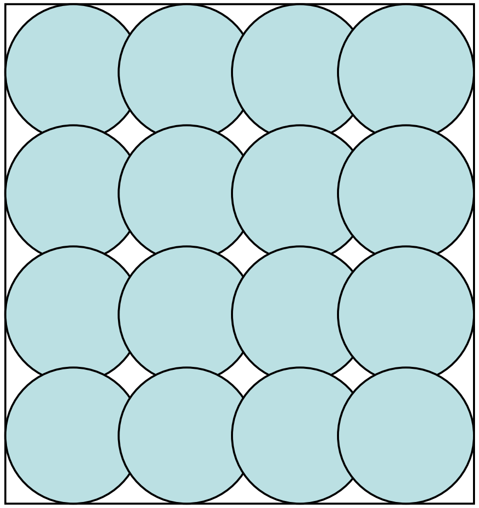
⇒ first analytic solution, then gradient descent starting from this solution

(gradient descent) requires differentiable basis functions!

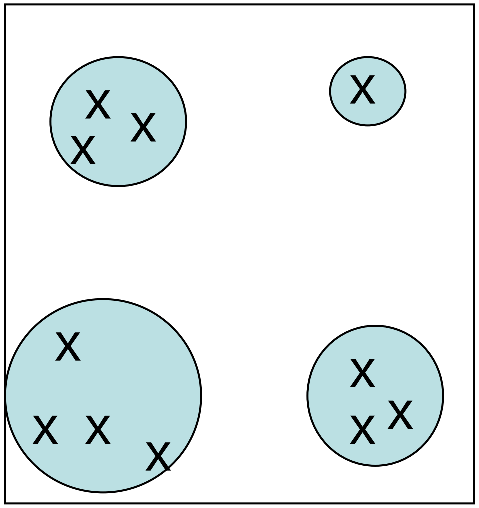
so far: tacitly assumed that RBF neurons are given

⇒ center c_k and radii σ considered given and known

how to choose c_k and σ ?



uniform covering



if training patterns inhomogenously distributed then first cluster analysis

choose center of basis function from each cluster, use cluster size for setting σ

advantages:

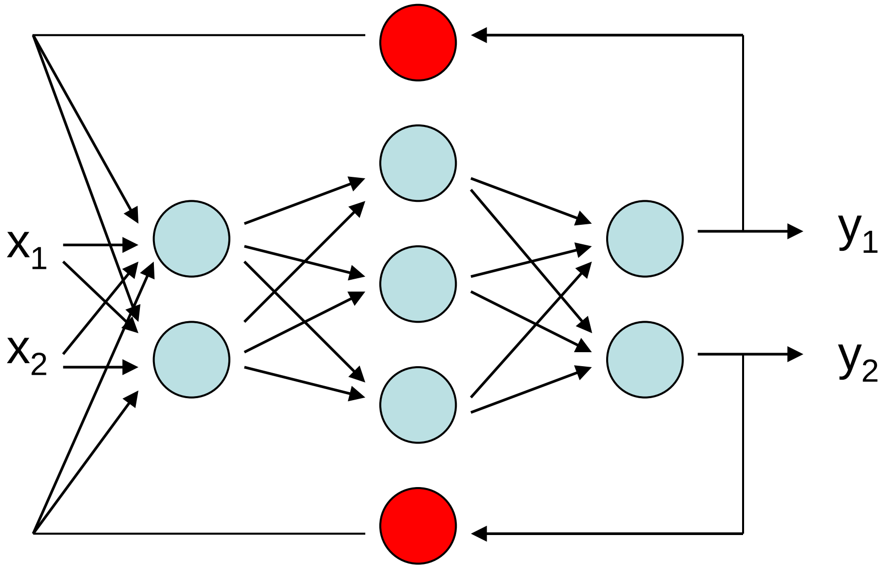
- additional training patterns → only local adjustment of weights
- optimal weights determinable in polynomial time
- regions not supported by RBF net can be identified by zero outputs

disadvantages:

- number of neurons increases exponentially with input dimension
- unable to extrapolate (since there are no centers and RBFs are local)

Jordan nets (1986)

- **context neuron**:
  reads output from some neuron at step t and feeds value into net at step t+1



Jordan net =

MLP + context neuron
for each output,
context neurons fully
connected to input layer

Elman nets (1990)

Elman net =

MLP + context neuron for each neuron output of MLP,
context neurons fully connected to associated MLP layer

Training?

⇒ unfolding in time (“loop unrolling“)

- identical MLPs serially connected (finitely often)
- results in a large MLP with many hidden (inner) layers
- backpropagation may take a long time
- but reasonable if most recent past more important than layers far away

Why using backpropagation?

⇒ use *Evolutionary Algorithms* directly on recurrent MLP!

later!