technische universität dortmund

Computational Intelligence

Winter Term 2010/11

Prof. Dr. Günter Rudolph
Lehrstuhl für Algorithm Engineering (LS 11)
Fakultät für Informatik
TU Dortmund

Plan for Today

- Application Fields of ANNs
  - Classification
  - Prediction
  - Function Approximation
- Radial Basis Function Nets (RBF Nets)
  - Model
  - Training
- Recurrent MLP
  - Elman Nets
  - Jordan Nets

Application Fields of ANNs

Classification

given: set of training patterns (input / output) — input $\tilde{x}_i$, output $\tilde{y}_i$

output = label
(e.g. class A, class B, ...)

$$f(\underbrace{x}_{\text{input (unknown)}}; \overbrace{\underbrace{(\tilde{x}_1, \tilde{y}_1), \ldots, (\tilde{x}_m, \tilde{y}_m)}_{\text{training patterns (known)}}, \underbrace{w_1, \ldots, w_n}_{\text{weights (learnt)}}}^{\text{parameters}}) \rightarrow \underbrace{\hat{y}}_{\text{output (guessed)}}$$

phase I:
train network

phase II:
apply network to unkown inputs for classification

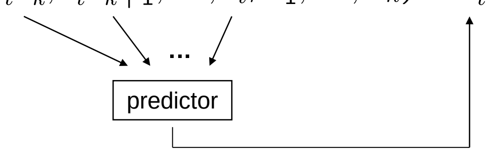
Application Fields of ANNs

Prediction of Time Series

time series $x_1, x_2, x_3, \ldots$ (e.g. temperatures, exchange rates, ...)

task: given a subset of historical data, predict the future

$$f(x_{t-k}, x_{t-k+1}, \ldots, x_t; w_1, \ldots, w_n) \rightarrow \hat{x}_{t+\tau}$$



training patterns:

historical data where true output is known;

error per pattern = $(\hat{x}_{t+\tau} - x_{t+\tau})^2$

phase I:
train network

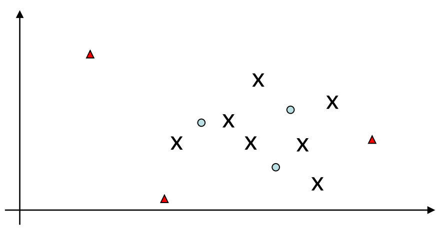
phase II:
apply network to historical inputs for predicting <u>unkown</u> outputs

Function Approximation (the general case)

task: given training patterns (input / output), approximate unkown function

→ should give outputs close to true unkown function for arbitrary inputs

- values between training patterns are **interpolated**
- values outside convex hull of training patterns are **extrapolated**



x : input training pattern

∘ : input pattern where output to be interpolated

▲ : input pattern where output to be extrapolated

Definition:
A function $\phi : \mathbb{R}^n \to \mathbb{R}$ is termed **radial basis function**
iff $\exists\, \varphi : \mathbb{R} \to \mathbb{R} : \forall\, x \in \mathbb{R}^n : \phi(x; c) = \varphi(\,\|x - c\|\,)$. □

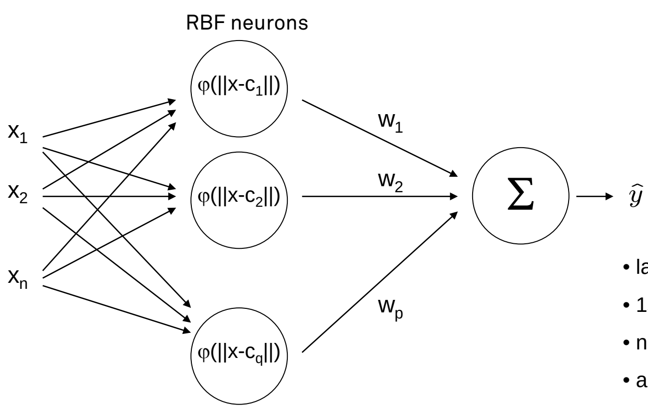
Definition:
RBF **local** iff
$\varphi(r) \to 0$ as $r \to \infty$ □

typically, $\|x\|$ denotes Euclidean norm of vector x

examples:

$\varphi(r) = \exp\left(-\frac{r^2}{\sigma^2}\right)$	Gaussian	unbounded	local
$\varphi(r) = \frac{3}{4}(1-r^2)\cdot 1_{\{r\le 1\}}$	Epanechnikov	bounded	local
$\varphi(r) = \frac{\pi}{4}\cos\left(\frac{\pi}{2}r\right)\cdot 1_{\{r\le 1\}}$	Cosine	bounded	local

Definition:
A function f: $\mathbb{R}^n \to \mathbb{R}$ is termed **radial basis function net (RBF net)**
iff $f(x) = w_1\, \varphi(\|x - c_1\|) + w_2\, \varphi(\|x - c_2\|) + \ldots + w_p\, \varphi(\|x - c_q\|)$ □



- layered net
- 1st layer fully connected
- no weights in 1st layer
- activation functions differ

given : N training patterns (x_i, y_i) and q RBF neurons

find : weights $w_1, \ldots, w_q$ with minimal error

solution:

we know that $f(x_i) = y_i$ for i = 1, ..., N or equivalently

$$\sum_{k=1}^{q} \underbrace{w_k}_{\text{unknown}} \cdot \underbrace{\varphi(\|x_i - c_k\|)}_{p_{ik}\ \text{known value}} = \underbrace{y_i}_{\text{known value}}$$

$$\Rightarrow \sum_{k=1}^{q} w_k \cdot p_{ik} = y_i$$

⇒ N linear equations with q unknowns

in matrix form: $P w = y$ with $P = (p_{ik})$ and P: N x q, y: N x 1, w: q x 1,

case $N = q$: $w = P^{-1} y$ if P has full rank

case $N < q$: many solutions but of no practical relevance

case $N > q$: $w = P^{+} y$ where P^{+} is Moore-Penrose pseudo inverse

$P w = y$ | · P‘ from left hand side (P‘ is transpose of P)

$P'P w = P' y$ | · $(P'P)^{-1}$ from left hand side

$(P'P)^{-1} P'P w = (P'P)^{-1} P' y$ | simplify

$\underbrace{(P'P)^{-1} P'P}_{\text{unit matrix}} w = \underbrace{(P'P)^{-1} P'}_{P^{+}} y$

complexity (naive)

$w = (P'P)^{-1} P' y$

$\underbrace{P'P: N^2 q \quad \text{inversion: } q^3 \quad P'y: qN \quad \text{multiplication: } q^2}_{O(N^2 q)}$

remark: if N large then inaccuracies for P‘P likely

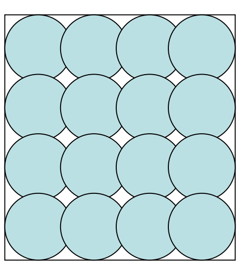
⇒ first analytic solution, then gradient descent starting from this solution

(gradient descent: requires differentiable basis functions!)

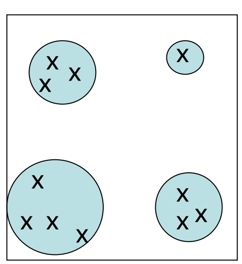
so far: tacitly assumed that RBF neurons are given

⇒ center c_k and radii σ considered given and known

how to choose c_k and σ ?



uniform covering



if training patterns inhomogenously distributed then first cluster analysis

choose center of basis function from each cluster, use cluster size for setting σ

advantages:

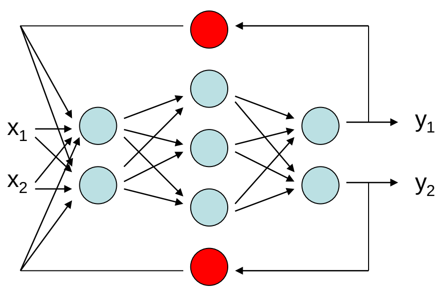
- additional training patterns → only local adjustment of weights
- optimal weights determinable in polynomial time
- regions not supported by RBF net can be identified by zero outputs

disadvantages:

- number of neurons increases exponentially with input dimension
- unable to extrapolate (since there are no centers and RBFs are local)

Jordan nets (1986)

- **context neuron**:
  reads output from some neuron at step t and feeds value into net at step t+1



Jordan net =

MLP + context neuron for each output, context neurons fully connected to input layer

Elman nets (1990)

Elman net =

MLP + context neuron for each neuron output of MLP,
context neurons fully connected to associated MLP layer

Training?

$\Rightarrow$ unfolding in time (“loop unrolling“)

- identical MLPs serially connected (finitely often)
- results in a large MLP with many hidden (inner) layers
- backpropagation may take a long time
- but reasonable if most recent past more important than layers far away

Why using backpropagation?

$\Rightarrow$ use *Evolutionary Algorithms* directly on recurrent MLP!

later!