A Framework of Quantum-inspired Multi-Objective Evolutionary Algorithms and its Convergence Condition

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ABSTRACT
A general framework of quantum-inspired multi-objective evolutionary algorithms as well as one of its sufficient convergence conditions to Pareto optimal set is proposed.

Categories and Subject Descriptors
F.1.2 [**Theory of Computation**] Modes of Computation — *Probabilistic computations.*

General Terms
Algorithms, Theory.

Keywords
Quantum computing, multi-objective evolutionary algorithms, Pareto optimal set, stochastic convergence.

1. INTRODUCTION
During the last two decades a large number of multi-objective evolutionary algorithms (MOEA) have been proposed, but only little theoretical work has been done[1, 2]. Recently, some specific algorithms combining MOEA with quantum computation principles, which are called quantum-inspired multi-objective evolutionary algorithms (QMOEA), were proposed[3]. Few theoretical results on the QMOEA have been done.

In this paper, we will propose a general framework of QMOEA, and give one of its sufficient convergence conditions to the Pareto optimal set. All definitions, preliminaries and proofs are omitted here due to limited space. They can be found in [4].

2. THE GENERAL FRAMEWORK OF QUANTUM-INSPIRED MULTIOBJECTIVE EVOLUTIONARY ALGORITHMS
Integrating the basic principles of QEA and general schemes of MOEA, we propose a general framework of quantum-inspired multi-objective evolutionary algorithms as follows:

The Procedure of the QMOEAs' Basic Framework

Begin
    $t \leftarrow 0$
i) Initialize $Q(t)$
ii) $A(t) = \{ \}$, $C(t) = \{ \}$
iii) While (not termination condition) do
        $t \leftarrow t+1$
iv) Make P(t) by observing the state of Q(t-1)
v) Evolve $P(t)$
vi) Make $C(t) = M_f(P(t) \bigcup C(t\text{-}1), \leq)$
        \\ ***Normally this step is eliminated.***
vii) Rebuild the archive set A(t)
        \\ ***Here*** $A(t) \subseteq M_f(P(t) \bigcup A(t\text{-}1), \leq)$ ***and maximize***
        \\ ***the diversity of those chosen elements in A(t).***
viii) Make $Q(t)$ by updating $Q(t\text{-}1)$ on Q-gates.
    End
End

3. ONE OF THE SUFFICIENT CONVERGENCE CONDITIONS
Theorem (Sufficient Convergence Condition) Let S be a feasible solution set of MOP. One of the sufficient conditions by whose this QMOEA converges with probability 1 to its Pareto optimal set is that there exists a real number ε_0,$0<\varepsilon_0<1$, which satisfies $P(s \in P(t)) \geq \varepsilon_0$ for all $s \in S$, $t > 0$ and $P(s \in P(t))$ is independent from each other for different t. Here $P(s \in P(t))$ denotes the probability that the population $P(t)$ contains s.

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