Problem Definitions for Performance Assessment on Multi-objective Optimization Algorithms

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Optimizing multiple conflicting objectives results in more than one optimal solution (known as Pareto-optimal solutions). Although only one of these solutions will be adopted at the end, the recent trend in evolutionary and classical multi-objective optimization studies have focused on approximating the set of Pareto-optimal solutions. It is believed that such a set of solutions will collectively provide good insight to the different trade-off regions on the Pareto-optimal front, thereby aiding a better and more confident decision making at the end. However, the type of Pareto-optimal approximation being sought strongly depends on the decision maker; here, aspects such as convergence to the Pareto-optimal front and the maintenance of solution diversity are important. Thus, to assess the performance of such optimization algorithms, the preferences of the decision maker must be taken into account.

Evolutionary Multi-objective Optimization (EMO) methodologies were suggested in the early 1990s for this task, and since then a number of performance assessment methods have been suggested. Most of the existing simulation studies that compare different EMO methodologies are based on a limited subset of performance measures. After more than 10 years of research and development into efficient EMO algorithms, the time is now ripe for the evaluation of existing EMO and classical generating methodologies using carefully selected test problems. The goal is to incorporate different consideration, formalized in terms of appropriate performance measures, so as to bring out the essential features required of an algorithm and thereby efficiently solve multi-objective optimization problems with due diligence given to the preferences of the decision maker. The comparisons will be made for a limited number of overall evaluations, to facilitate the evaluation of existing or new algorithms with respect to different functional abilities, namely: i) to meet well specified preferences (convergence to the Pareto front, diversity, objective values, etc.) ii) to scale well on many objectives, and iii) to scale well on many variables.

In section 1, we specify a set of test functions that capture different, commonly considered preference types as per the recent literature. Section 2 and 3 provide performance assessment and algorithm complexity guidelines. A suggested format for presenting results is shown in the last section.

1 Definitions of the Test Suite

In this section, 13 multi-objective optimization problems are described. We assume that the optimization problems under consideration involve M objective functions $f_1, \dots, f_M$ that are all to be minimized.

First we choose test functions OKA2 [3] and SYMPART [6] proposed recently. Second, we select some problems from two immensely popular test suites, ZDT and DTLZ [1][2]. Finally three test functions of the newly proposed WFG test suite are included.[4]

However, the ZDT and DTLZ test suites have the following problems:

- For all problems, the global optimum has the same parameter values for different variables/dimensions
- The global optimum lies in the center of the search range

- The global optimum lies on the bounds
- All of these problems are separable

To overcome these shortcomings, we shifted or rotated the original ZDT and DTLZ problem.

❑ Extending and Shifting

*f(**z**)*: original function. Search range [**zmin** , **zmax**]

*F(**x**)*: new extended function. Search range [**xmin**, x**max**]

D: dimension

$\mathbf{o} = [o_1, o_2, \ldots o_D]$: the shifted vector in parameter space

$\mathbf{d} = [d_1, d_2, ...d_D]$: the extended length of the lower bound

$\boldsymbol{\lambda} = [\lambda_1, \lambda_2, ...\lambda_D]$: the scale factor

$\mathbf{p} = [p_1, p_1, ...p_D]$: the penalty value

1) To overcome the shortcomings of the ZDT test functions for which the global optimum lies on the lower bound, or in the center of the search range, we extend the lower bound **zmin** by **d**. Then, for the solution in the extended region, the function value is obtained by mapping and stretching.

$$f'(\mathbf{z}') = S(psum)\left(f(\mathbf{z}') + f\_bias\right)$$

where $z_i' = \begin{cases} z_i, & \mathrm{z}_i \ge z\min_i \\ z\min_i + \lambda_i (z\min_i - z_i), & \mathrm{z}_i < z\min_i \end{cases}$,

$S(psum) = \dfrac{2}{1+\exp(-psum)}$, $psum = \sqrt{\sum_{i\in \mathrm{I}} p_i^2}, I \subseteq \{1, 2, ...D\}$ (I is a set of all variables included in the objective function *f(x)*)

Here the constant parameter vector $\boldsymbol{\lambda}$ is used to make the searching region not symmetric with respect to the variable.

Here the stretching function **S** is used to guarantee that the objective function values of solutions in the extended region are always worse than those in the original region, i.e., the Pareto Optimal front remains unchanged. This assumption holds true on the condition that *f>0*. Therefore we shift f to $f + f\_{bias}$ to make sure that all function values are positive. The range of the function S is [1, 2]. When one solution in the extended region is near the mapping center, there will be *psum*→0 and S→1. On the contrary, if the solution is far from the mapping center, S→2. Thus we enlarge the objective value in the extended region whilst at the same time keeping the function connected.

The penalty value p_i in each variable is calculated as:

$$p_i = \begin{cases} 0, & \mathrm{z}_i \ge z\min_i \\ |z\min_i - z_i| / d_i, & \mathrm{z}_i < z\min_i \end{cases}, \quad i = 1, 2, ..., D$$

2) After extending the region, shift the parameter space by vector **o**, and then the new function $F(\mathbf{x}) = f'(\mathbf{x}),\ \mathbf{x} = \mathbf{z} + \mathbf{o}$

Therefore, the extended shifted function will be:

$$F_m(\mathbf{x}) = \begin{cases} f_m(\mathbf{z}')+1 & \text{for all } z_i \ge z\min_i \\ S(psum_m)\left(f_m(\mathbf{z}')+1\right) & \text{otherwise} \end{cases}, \quad m = 1,2,...M \;\; , \; \mathbf{z} = \mathbf{x}-\mathbf{o}$$

where $S(psum) = \dfrac{2}{1+\exp(-psum)}$, $psum = \sqrt{\sum_{i\in I} p_i^2}, I \subseteq \{1,2,...D\}$

$$z_i' = \begin{cases} z_i, & z_i \ge z\min_i \\ z\min_i + \lambda_i(z\min_i - z_i), & z_i < z\min_i \end{cases} \qquad p_i = \begin{cases} 0, & z_i \ge z\min_i \\ |z\min_i - z_i| / d_i, & z_i < z\min_i \end{cases}, \quad i = 1,2,...,D$$

According to the above description, we extended and shifted ZDT1, ZDT2, ZDT4, ZDT6, DTLZ2, DTLZ3, obtaining S_ZDT1, S_ZDT2, S_ZDT4, S_ZDT6, S_DTLZ2, S_DTLZ3.

❑ Extending and Rotation

The detailed principle of extend are the same as the previous description.

$$F_m(\mathbf{x}) = \begin{cases} f_m(\mathbf{z}')+1 & \text{for all } x\min_i \le x_i \le x\max_i \\ S(psum_m)\left(f_m(\mathbf{z}')+1\right) & \text{otherwise} \end{cases}, \quad m = 1,2,...M \;\; , \; \mathbf{z} = \mathbf{M}\mathbf{x}$$

where $S(psum) = \dfrac{2}{1+\exp(-psum)}$, $psum = \sqrt{\sum_{i\in I} p_i^2}, I \subseteq \{1,2,...D\}$

$$z_i' = \begin{cases} z\min_i + \lambda_i(z\min_i - z_i), & z_i < z\min_i \\ z_i, & z\min_i \le z_i \le z\max_i \\ z\max_i - \lambda_i(z_i - z\max_i), & z_i > z\max_i \end{cases} \qquad p_i = \begin{cases} z\min_i - z_i, & z_i < z\min_i \\ 0, & z\min_i \le z_i \le z\max_i \\ z_i - z\max_i, & z_i > z\max_i \end{cases}$$

$i = 1,2,...,D$

M: linear transformation orthogonal matrix, with condition number=1.

According to the above description, we extended and rotated ZDT4, DTLZ2, obtaining R_ZDT4, R_DTLZ2.

1) OKA2

$$f_1(\mathbf{x}) = x_1$$
$$f_2(\mathbf{x}) = 1 - \frac{1}{4\pi^2}(x_1+\pi)^2 + |x_2 - 5\cos(x_1)|^{\frac{1}{3}} + |x_3 - 5\sin(x_1)|^{\frac{1}{3}}$$

$x_1 \in [-\pi, \pi]$, $x_2, x_3 \in [-5, 5]$

Pareto Set $(x_1, x_2, x_3) = (\xi, 5\cos(\xi), 5\sin(\xi))$, $\xi \in [-\pi, \pi]$

2) SYM-PART

$$f_1(\mathbf{x}) = (x_1' + a - t_1 c_2)^2 + (x_2' - t_2 b)^2 + ... + (x_{D-1}' + a - t_1 c_2)^2 + (x_D' - t_2 b)^2$$
$$f_2(\mathbf{x}) = (x_1' - a - t_1 c_2)^2 + (x_2' - t_2 b)^2 + ... + (x_{D-1}' - a - t_1 c_2)^2 + (x_D' - t_2 b)^2$$

where

$$\mathbf{x}' = \begin{pmatrix} \cos\omega & -\sin\omega & 0 & 0 & ... \\ \sin\omega & \cos\omega & 0 & 0 & ... \\ 0 & 0 & \cos\omega & -\sin\omega & ... \\ 0 & 0 & \sin\omega & \cos\omega & ... \\ ... & ... & ... & ... & ... \end{pmatrix} \mathbf{x}$$

$$t_1 = \text{sgn}(x_1') \times \left\lceil \frac{|x_1'| - c_2/2}{c_2} \right\rceil, t_2 = \text{sgn}(x_2') \times \left\lceil \frac{|x_2'| - b/2}{b} \right\rceil$$

$a = 1, b = 10, c = 8, c_2 = c + 2a = 10;$

D : dimension

$\mathbf{x} \in [-20, 20]^D$

3) Extended shifted ZDT1 (S_ZDT1)

$$f_1(x) = \begin{cases} z_1' + 1, & z_1 \ge 0 \\ S(p_1)(z_1' + 1), & z_1 < 0 \end{cases}$$

$$f_2(x) = \begin{cases} g(x)[1 - \sqrt{z_1' / g(x)}] + 1, & \text{all } z_i \ge 0 \\ S(\sqrt{\sum_{i=1}^{D} p_i^2})\left(g(x)[1 - \sqrt{z_1' / g(x)}] + 1\right), & \textit{otherwise} \end{cases}$$

$$g(x) \quad = 1 + 9 \cdot (\textstyle\sum_{i=2}^{D} z_i')/(D-1)$$

$$S(t) = \frac{2}{1 + \exp(-t)}$$

where $z_i' = \begin{cases} z_i, & z_i \ge 0 \\ -\lambda_i z_i, & z_i < 0 \end{cases}$, $p_i = \begin{cases} 0, & z_i \ge 0 \\ |z_i| / d_i, & z_i < 0 \end{cases}$, $i = 1, 2, ..., D$

$\mathbf{z} = \mathbf{x} - \mathbf{o}, \quad \mathbf{x} = [x_1, x_2, \ldots x_D], \mathbf{z} = [z_1, z_2, \ldots z_D]$

D : dimension

$\mathbf{o} = [o_1, o_2, \ldots o_D]$: the shifted vector in parameter space

$\mathbf{d} = [d_1, d_2, ...d_D]$: the extended length of the lower bound

$\boldsymbol{\lambda} = [\lambda_1, \lambda_2, ...\lambda_D]$: the scale factor

$\mathbf{p} = [p_1, p_2, ...p_D]$: the penalty value

$x_i \in [x\min_i, x\max_i]$, $\mathbf{xmin} = [x\min_1, x\min_2, ...x\min_D]$ and $\mathbf{xmax} = [x\max_1, x\max_2, ...x\max_D]$

The optimal solutions: $x_1 \in [o_1, \ 1\text{+o}_1]$ and $x_i = o_i, \ \ i = 2, \ldots, D$

Data file:

Name	Variable	Note
S_ZDT1.dat	3*100 vector 1st row: **o** 2nd row: **d** 3rd row: λ	When using, cut **o**=**o**(1:D) **d**=**d**(1:D) λ=λ (1:D) **xmin**=**xmin**(1:D) **xmax**=**xmax**(1:D)
S_ZDT1_bound.dat	2*100 matrix 1st row: **xmin** 2nd row: **xmax**	

4) Extended Shifted ZDT2 (S_ ZDT2)

$$f_1(x)=\begin{cases} z_1'+1, & z_1\ge 0 \\ S(p_1)(z_1'+1), & z_1<0 \end{cases}$$

$$f_2(x)=\begin{cases} g(x)[1-(z_1'/g(x))^2]+1, & z_i\ge 0 \\ S\left(\sqrt{\sum_{i=1}^{D}p_i^2}\right)\left(g(x)[1-(z_1'/g(x))^2]+1\right), & otherwise \end{cases}$$

$$g(x) \quad =1+9\cdot(\textstyle\sum_{i=2}^{D}z_i')/(D-1)$$

where $z_i'=\begin{cases} z_i, & z_i\ge 0 \\ -\lambda_i z_i, & z_i<0 \end{cases}$, $p_i=\begin{cases} 0, & z_i\ge 0 \\ |z_i|/d_i, & z_i<0 \end{cases}$, $i=1,2,...,D$,

$\mathbf{z}=\mathbf{x}-\mathbf{o}$, $\mathbf{x}=[x_1,x_2,\dots x_D]$, $\mathbf{z}=[z_1,z_2,\dots z_D]$

D : dimension

$\mathbf{o}=[o_1,o_2,\dots o_D]$: the shifted vector in parameter space

$\mathbf{d}=[d_1,d_2,...d_D]$: the extended length of the lower bound

$\boldsymbol{\lambda}=[\lambda_1,\lambda_2,...\lambda_D]$: the scale factor

$\mathbf{p}=[p_1,p_1,...p_D]$: the penalty value

$x_i\in[x\min_i, x\max_i]$, $\mathbf{xmin}=[x\min_1,x\min_2,...x\min_D]$ and $\mathbf{xmax}=[x\max_1,x\max_2,...x\max_D]$

The optimal solutions $x_1\in[o_1,\ 1+o_1]$ and $x_i=o_i,\ \ i=2,\dots,D$

Data file:

Name	Variable	Note
S_ZDT2.dat	3*100 vector 1st row: **o** 2nd row: **d** 3rd row: λ	When using, cut **o**=**o**(1:D) **d**=**d**(1:D) λ=λ (1:D) **xmin**=**xmin**(1:D) **xmax**=**xmax**(1:D)
S_ZDT2_bound.dat	2*100 matrix 1st row: **xmin** 2nd row: **xmax**	

5) Extended Shifted ZDT4 (S_ZDT4)

$$f_1(x)=\begin{cases} z_1'+1, & z_1\ge 0 \\ S(p_1)(z_1'+1), & z_1<0 \end{cases}$$

$$f_2(x)=\begin{cases} g(x)[1-\sqrt{z_1'/g(x)}]+1, & \text{all } z_i\ge -5 \\ S\left(\sqrt{\sum_{i=1}^{D}p_i^2}\right)\left(g(x)[1-\sqrt{z_1'/g(x)}]+1\right), & otherwise \end{cases}$$

$$g(x) \quad =1+10(D-1)+\textstyle\sum_{i=2}^{D}[z_i'^2-10\cos(4\pi z_i')]$$

where $z_1'=\begin{cases} z_1, & z_1\ge 0 \\ -\lambda_1 z_1, & z_1<0 \end{cases}$, $p_1=\begin{cases} 0, & z_1\ge 0 \\ |z_1|/d_1, & z_1<0 \end{cases}$

$$z_i' = \begin{cases} z_i, & z_i \geq -5 \\ -5-\lambda_i(z_i+5), & z_i < -5 \end{cases}, \quad p_i = \begin{cases} 0, & z_i \geq -5 \\ |z_i+5|/d_i, & z_i < -5 \end{cases}, \quad i = 2,...,D$$

$\mathbf{z} = \mathbf{x} - \mathbf{o}$, $\mathbf{x} = [x_1, x_2, \ldots x_D], \mathbf{z} = [z_1, z_2, \ldots z_D]$

D : dimension

$\mathbf{o} = [o_1, o_2, \ldots o_D]$: the shifted vector in parameter space

$\mathbf{d} = [d_1, d_2, ...d_D]$: the extended length of the lower bound

$\boldsymbol{\lambda} = [\lambda_1, \lambda_2, ...\lambda_D]$: the scale factor

$\mathbf{p} = [p_1, p_1, ...p_D]$: the penalty value

$x_i \in [x\min_i, x\max_i]$, $\mathbf{xmin} = [x\min_1, x\min_2, ...x\min_D]$ and $\mathbf{xmax} = [x\max_1, x\max_2, ...x\max_D]$ are included in the data file.

The optimal solutions $x_1 \in [o_1, 1+o_1]$ and $x_i = o_i$, $i = 2, \ldots, D$

Data file:

Name	Variable	Note
S_ZDT4.dat	3*100 vector 1st row: **o** 2nd row: **d** 3rd row: **λ**	When using, cut **o**=**o**(1:D) **d**=**d**(1:D) **λ**=**λ** (1:D)
S_ZDT4_bound.dat	2*100 matrix 1st row: **xmin** 2nd row: **xmax**	**xmin**=**xmin**(1:D) **xmax**=**xmax**(1:D)

6) Extended rotated ZDT4 (R_ZDT4)

$$f_1(x) = \begin{cases} z_1' + 1, & z_1 \geq 0 \\ S(p_1)(z_1' + 1), & z_1 < 0 \end{cases}$$

$$f_2(x) = \begin{cases} g(x)[1 - \sqrt{z_1'/g(x)}] + 1, & \text{all } z_i \geq -5 \\ S\left(\sqrt{\sum_{i=1}^{D} p_i^2}\right)\left(g(x)[1 - \sqrt{z_1'/g(x)}] + 1\right), & \text{otherwise} \end{cases}$$

$$g(x) = 1 + 10(D-1) + \sum_{i=2}^{D} [z_i'^2 - 10\cos(4\pi z_i')]$$

where $$z_1' = \begin{cases} -\lambda_1 z_1, & z_1 < 0 \\ z_1, & 0 \leq z_1 \leq 1 \\ \lambda_1 z_1, & z_1 > 1 \end{cases}, \quad p_i = \begin{cases} -z_1, & z_1 < 0 \\ 0, & 0 \leq z_1 \leq 1 \\ z_1 - 1, & z_1 > 1 \end{cases}$$

$$z_i' = \begin{cases} -5-\lambda_i(z_i+5), & z_i < -5 \\ z_i, & -5 \leq z_i \leq 5 \\ 5-\lambda_i(z_i-5), & z_i > 5 \end{cases}, \quad p_i = \begin{cases} -5-z_i, & z_i < -5 \\ 0, & -5 \leq z_i \leq 5 \\ z_i - 5, & z_i > 5 \end{cases}, \quad i = 2,...,D$$

$\mathbf{z} = \mathbf{Mx}$, $\mathbf{x} = [x_1, x_2, \ldots x_D], \mathbf{z} = [z_1, z_2, \ldots z_D]$

D : dimension

$\boldsymbol{\lambda} = [\lambda_1, \lambda_2, ...\lambda_D]$: the scale factor

$\mathbf{p} = [p_1, p_1, ...p_D]$: the penalty value

$x_i \in [x\min_i, x\max_i]$, $\mathbf{xmin} = [x\min_1, x\min_2, ...x\min_D]$ and $\mathbf{xmax} = [x\max_1, x\max_2, ...x\max_D]$

Data file:

10D

Name	Variable
R_ZDT4_M_10D.dat	**M** 10*10 matrix
R_ZDT4_bound_10D.dat	2*10 matrix 1st row: **xmin** 2nd row: **xmax**
R_ZDT4_lamda_10D.dat	λ 1*10D vector

7) Extended shifted ZDT6 (S_ZDT6)

$$f_1(x) = \begin{cases} 1 - \exp(-4z_1')\sin^6(6\pi z_1') + 1, & z_1 \ge 0 \\ S(p_1)\left(1 - \exp(-4z_1')\sin^6(6\pi z_1') + 1\right), & z_1 < 0 \end{cases}$$

$$f_2(x) = \begin{cases} g(x)[1 - (z_1' / g(x))^2] + 1, & z_i \ge 0 \\ S(\sqrt{\sum_{i=1}^{D} p_i^2})\left(g(x)[1 - (z_1' / g(x))^2] + 1\right), & otherwise \end{cases}$$

$$g(x) \quad = 1 + 9\left[(\sum_{i=2}^{D} z_i') / (D-1)\right]^{0.25}$$

where $z_i' = \begin{cases} z_i, & z_i \ge 0 \\ -\lambda_i z_i, & z_i < 0 \end{cases}$, $p_i = \begin{cases} 0, & z_i \ge 0 \\ |z_i| / d_i, & z_i < 0 \end{cases}$, $i = 1, 2, ..., D$

$\mathbf{z} = \mathbf{x} - \mathbf{o}$, $\mathbf{x} = [x_1, x_2, \dots x_D], \mathbf{z} = [z_1, z_2, \dots z_D]$

D : dimension

$\mathbf{o} = [o_1, o_2, \dots o_D]$: the shifted vector in parameter space

$\mathbf{d} = [d_1, d_2, ...d_D]$: the extended length of the lower bound

$\boldsymbol{\lambda} = [\lambda_1, \lambda_2, ...\lambda_D]$: the scale factor

$\mathbf{p} = [p_1, p_1, ...p_D]$: the penalty value

$x_i \in [x\min_i, x\max_i]$, $\mathbf{xmin} = [x\min_1, x\min_2, ...x\min_D]$ and $\mathbf{xmax} = [x\max_1, x\max_2, ...x\max_D]$

The optimal solutions: $x_1 \in [o_1,\ 1+o_1]$ and $x_i = o_i$, $i = 2, \dots, D$

Data file:

Name	Variable	Note
S_ZDT6.dat	3*100 vector 1st row: **o** 2nd row: **d** 3rd row: **λ**	When using, cut **o**=**o**(1:D) **d**=**d**(1:D) **λ**=**λ** (1:D)
S_ZDT6_bound.dat	2*100 matrix 1st row: **xmin** 2nd row: **xmax**	**xmin**=**xmin**(1:D) **xmax**=**xmax**(1:D)

8) Extended shifted DTLZ2 (S_DTLZ2)

$$f_1(\mathbf{x}) = \begin{cases} (1+g(\mathbf{x}_M))\cos\left(z_1'\,\pi/2\right)\cos\left(z_2'\,\pi/2\right)\ldots\cos\left(z_{M-2}'\,\pi/2\right)\cos\left(z_{M-1}'\,\pi/2\right)+1, & z_i \ge 0 \\ S(psum_1)\left((1+g(\mathbf{x}_M))\cos\left(z_1'\,\pi/2\right)\cos\left(z_2'\,\pi/2\right)\ldots\cos\left(z_{M-2}'\,\pi/2\right)\cos\left(z_{M-1}'\,\pi/2\right)+1\right), & \text{otherwise} \end{cases}$$

$$f_2(\mathbf{x}) = \begin{cases} (1+g(\mathbf{x}_M))\cos\left(z_1'\,\pi/2\right)\cos\left(z_2'\,\pi/2\right)\ldots\cos\left(z_{M-2}'\,\pi/2\right)\sin\left(z_{M-1}'\,\pi/2\right)+1, & z_i \ge 0 \\ S(psum_2)\left((1+g(\mathbf{x}_M))\cos\left(z_1'\,\pi/2\right)\cos\left(z_2'\,\pi/2\right)\ldots\cos\left(z_{M-2}'\,\pi/2\right)\sin\left(z_{M-1}'\,\pi/2\right)+1\right), & \text{otherwise} \end{cases}$$

$$f_3(\mathbf{x}) = \begin{cases} (1+g(\mathbf{x}_M))\cos\left(z_1'\,\pi/2\right)\cos\left(z_2'\,\pi/2\right)\ldots\sin\left(z_{M-2}'\,\pi/2\right)+1, & z_i \ge 0 \\ S(psum_3)\left((1+g(\mathbf{x}_M))\cos\left(z_1'\,\pi/2\right)\cos\left(z_2'\,\pi/2\right)\ldots\sin\left(z_{M-2}'\,\pi/2\right)+1\right), & \text{otherwise} \end{cases}$$

$$\vdots$$

$$f_{M-1}(\mathbf{x}) = \begin{cases} (1+g(\mathbf{x}_M))\cos\left(z_1'\,\pi/2\right)\sin\left(z_2'\,\pi/2\right)+1, & z_i \ge 0 \\ S(psum_{M-1})\left((1+g(\mathbf{x}_M))\cos\left(z_1'\,\pi/2\right)\sin\left(z_2'\,\pi/2\right)+1\right), & \text{otherwise} \end{cases}$$

$$f_M(\mathbf{x}) = \begin{cases} (1+g(\mathbf{x}_M))\sin\left(z_1'\,\pi/2\right)+1, & z_i \ge 0 \\ S(psum_M)\left((1+g(\mathbf{x}_M))\sin\left(z_1'\,\pi/2\right)+1\right), & \text{otherwise} \end{cases}$$

$$g(\mathbf{x}_M) = \sum_{x_i \in \mathbf{x}_M} \left(z_i' - 0.5\right)^2$$

where $z_i' = \begin{cases} z_i, & z_i \ge 0 \\ -\lambda_i z_i, & z_i < 0 \end{cases}$, $p_i = \begin{cases} 0, & z_i \ge 0 \\ |z_i|/d_i, & z_i < 0 \end{cases}$, $i = 1, 2, \ldots, D$

$\mathbf{z} = \mathbf{x} - \mathbf{o}$, $\mathbf{x} = [x_1, x_2, \ldots x_D], \mathbf{z} = [z_1, z_2, \ldots z_D]$

D : dimension

$\mathbf{o} = [o_1, o_2, \ldots o_D]$: the shifted vector in parameter space

$\mathbf{d} = [d_1, d_2, ...d_D]$: the extended length of the lower bound

$\boldsymbol{\lambda} = [\lambda_1, \lambda_2, ...\lambda_D]$: the scale factor

$\mathbf{p} = [p_1, p_1, ...p_D]$: the penalty value

$x_i \in [x\min_i, x\max_i]$, $\mathbf{xmin} = [x\min_1, x\min_2, ...x\min_D]$ and $\mathbf{x}\max = [x\max_1, x\max_2, ...x\max_D]$ are included in the data file.

The Pareto-optimal solutions $x_i^* = 0.5 + o_i \left(x_i^* \in \mathbf{x}_M\right)$ and the objective function values must satisfy: $\sum_{m=1}^{M}\left(f_m^*\right)^2 = 0.5$

Data file:

Name	Variable	Note
S_DTLZ2.dat	3*30 vector 1st row: **o** 2nd row: **d** 3rd row: **λ**	When using, cut **o**=**o**(1:D) **d**=**d**(1:D) λ=λ (1:D)
S_DTLZ2_bound.dat	2*30 matrix 1st row: **xmin** 2nd row: **xmax**	**xmin**=**xmin**(1:D) **xmax**=**xmax**(1:D)

9) Extended Rotated DTLZ2 (R_DTLZ2)

$$f_1(\mathbf{x}) = \begin{cases} (1+g(\mathbf{x}_M))\cos\left(z_1'\,\pi/2\right)\cos\left(z_2'\,\pi/2\right)\ldots\cos\left(z_{M-2}'\,\pi/2\right)\cos\left(z_{M-1}'\,\pi/2\right)+1, & z_i \ge 0 \\ S(psum_1)\left((1+g(\mathbf{x}_M))\cos\left(z_1'\,\pi/2\right)\cos\left(z_2'\,\pi/2\right)\ldots\cos\left(z_{M-2}'\,\pi/2\right)\cos\left(z_{M-1}'\,\pi/2\right)+1\right), & \text{otherwise} \end{cases}$$

$$f_2(\mathbf{x}) = \begin{cases} (1+g(\mathbf{x}_M))\cos\left(z_1'\,\pi/2\right)\cos\left(z_2'\,\pi/2\right)\ldots\cos\left(z_{M-2}'\,\pi/2\right)\sin\left(z_{M-1}'\,\pi/2\right)+1, & z_i \ge 0 \\ S(psum_2)\left((1+g(\mathbf{x}_M))\cos\left(z_1'\,\pi/2\right)\cos\left(z_2'\,\pi/2\right)\ldots\cos\left(z_{M-2}'\,\pi/2\right)\sin\left(z_{M-1}'\,\pi/2\right)+1\right), & \text{otherwise} \end{cases}$$

$$f_3(\mathbf{x}) = \begin{cases} (1+g(\mathbf{x}_M))\cos\left(z_1'\,\pi/2\right)\cos\left(z_2'\,\pi/2\right)\ldots\sin\left(z_{M-2}'\,\pi/2\right)+1, & z_i \ge 0 \\ S(psum_3)\left((1+g(\mathbf{x}_M))\cos\left(z_1'\,\pi/2\right)\cos\left(z_2'\,\pi/2\right)\ldots\sin\left(z_{M-2}'\,\pi/2\right)+1\right), & \text{otherwise} \end{cases}$$

$$\vdots$$

$$f_{M-1}(\mathbf{x}) = \begin{cases} (1+g(\mathbf{x}_M))\cos\left(z_1'\,\pi/2\right)\sin\left(z_2'\,\pi/2\right)+1, & z_i \ge 0 \\ S(psum_{M-1})\left((1+g(\mathbf{x}_M))\cos\left(z_1'\,\pi/2\right)\sin\left(z_2'\,\pi/2\right)+1\right), & \text{otherwise} \end{cases}$$

$$f_M(\mathbf{x}) = \begin{cases} (1+g(\mathbf{x}_M))\sin\left(z_1'\,\pi/2\right)+1, & z_i \ge 0 \\ S(psum_M)\left((1+g(\mathbf{x}_M))\sin\left(z_1'\,\pi/2\right)+1\right), & \text{otherwise} \end{cases}$$

$$g\left(\mathbf{x}_M\right) = \sum_{x_i \in \mathbf{x}_M}\left(z_i' - 0.5\right)^2$$

where $z_i' = \begin{cases} -\lambda_i z_i, & z_i < 0 \\ z_i, & 0 \le z_i \le 1 \\ \lambda_i z_i, & z_i > 1 \end{cases}$, $p_i = \begin{cases} -z_i, & z_i < 0 \\ 0, & 0 \le z_i \le 1, \\ z_i - 1, & z_i > 1 \end{cases}$ $i = 1, 2, ..., D$

$\mathbf{z} = \mathbf{M}\mathbf{x}$, $\mathbf{x} = [x_1, x_2, \ldots x_D], \mathbf{z} = [z_1, z_2, \ldots z_D]$

D : dimension

$\mathbf{o} = [o_1, o_2, \ldots o_D]$: the shifted vector in parameter space

$\boldsymbol{\lambda} = [\lambda_1, \lambda_2, ...\lambda_D]$: the scale factor

$\mathbf{p} = [p_1, p_1, ...p_D]$: the penalty value

$x_i \in [x\min_i, x\max_i]$, $\mathbf{xmin} = [x\min_1, x\min_2, ...x\min_D]$ and $\mathbf{x}\max = [x\max_1, x\max_2, ...x\max_D]$

Data file:

30D

Name	Variable
R_ DTLZ2_M_30D.dat	**M** 30*30 matrix
R_ DTLZ2_bound_30D.dat	2*30 matrix 1st row: **xmin** 2nd row: **xmax**
R_ DTLZ2_lamda_30D.dat	**λ** 1*30D vector

10) Extended shifted DTLZ3 (S_DTLZ3)

$$f_1(\mathbf{x})=\begin{cases}(1+g(\mathbf{x}_M))\cos(z_1'\,\pi/2)\cos(z_2'\,\pi/2)\ldots\cos(z_{M-2}'\,\pi/2)\cos(z_{M-1}'\,\pi/2), & z_i\ge 0\\ S(psum_1)\left((1+g(\mathbf{x}_M))\cos(z_1'\,\pi/2)\cos(z_2'\,\pi/2)\ldots\cos(z_{M-2}'\,\pi/2)\cos(z_{M-1}'\,\pi/2)+1\right), & \text{otherwise}\end{cases}$$

$$f_2(\mathbf{x})=\begin{cases}(1+g(\mathbf{x}_M))\cos(z_1'\,\pi/2)\cos(z_2'\,\pi/2)\ldots\cos(z_{M-2}'\,\pi/2)\sin(z_{M-1}'\,\pi/2), & z_i\ge 0\\ S(psum_2)\left((1+g(\mathbf{x}_M))\cos(z_1'\,\pi/2)\cos(z_2'\,\pi/2)\ldots\cos(z_{M-2}'\,\pi/2)\sin(z_{M-1}'\,\pi/2)+1\right), & \text{otherwise}\end{cases}$$

$$f_3(\mathbf{x})=\begin{cases}(1+g(\mathbf{x}_M))\cos(z_1'\,\pi/2)\cos(z_2'\,\pi/2)\ldots\sin(z_{M-2}'\,\pi/2), & z_i\ge 0\\ S(psum_3)\left((1+g(\mathbf{x}_M))\cos(z_1'\,\pi/2)\cos(z_2'\,\pi/2)\ldots\sin(z_{M-2}'\,\pi/2)+1\right), & \text{otherwise}\end{cases}$$

$$\vdots$$

$$f_{M-1}(\mathbf{x})=\begin{cases}(1+g(\mathbf{x}_M))\cos(z_1'\,\pi/2)\sin(z_2'\,\pi/2), & z_i\ge 0\\ S(psum_{M-1})\left((1+g(\mathbf{x}_M))\cos(z_1'\,\pi/2)\sin(z_2'\,\pi/2)+1\right), & \text{otherwise}\end{cases}$$

$$f_M(\mathbf{x})=\begin{cases}(1+g(\mathbf{x}_M))\sin(z_1'\,\pi/2), & z_i\ge 0\\ S(psum_M)\left((1+g(\mathbf{x}_M))\sin(z_1'\,\pi/2)+1\right), & \text{otherwise}\end{cases}$$

where $g(\mathbf{x}_M)=100\left(|\mathbf{x}_M|+\sum_{x_i\in\mathbf{x}_M}(z_i-0.5)^2-\cos(20\pi(z_i-0.5))\right)$

$$z_i'=\begin{cases}z_i, & z_i\ge 0\\ -\lambda_i z_i, & z_i<0\end{cases},\quad p_i=\begin{cases}0, & z_i\ge 0\\ |z_i|/d_i, & z_i<0\end{cases},\quad i=1,2,\ldots,D$$

$$\mathbf{z}=\mathbf{x}-\mathbf{o},\quad \mathbf{x}=[x_1,x_2,\ldots x_D],\mathbf{z}=[z_1,z_2,\ldots z_D]$$

D : dimension

$\mathbf{o}=[o_1,o_2,\ldots o_D]$: the shifted vector in parameter space

$\mathbf{d}=[d_1,d_2,\ldots d_D]$: the extended length of the lower bound

$\boldsymbol{\lambda}=[\lambda_1,\lambda_2,\ldots\lambda_D]$: the scale factor

$\mathbf{p}=[p_1,p_1,\ldots p_D]$: the penalty value

$x_i\in[x\min_i,x\max_i]$, $\mathbf{xmin}=[x\min_1,x\min_2,\ldots x\min_D]$ and $\mathbf{xmax}=[x\max_1,x\max_2,\ldots x\max_D]$ are included in the data file.

The Pareto-optimal solutions $x_i^*=0.5+o_i\,(x_i^*\in\mathbf{x}_M)$.

Data file:

Name	Variable	Note
S_DTLZ3.dat	3*30 vector 1st row: **o** 2nd row: **d** 3rd row: λ	When using, cut **o**=**o**(1:D) **d**=**d**(1:D) λ=λ (1:D) **xmin**=**xmin**(1:D) **xmax**=**xmax**(1:D)
S_DTLZ3_bound.dat	2*30 matrix 1st row: **xmin** 2nd row: **xmax**	

WFG [4]

Given $\mathbf{z} = \{z_1, ..., z_k, z_{k+1}, ..., z_n\}$

Minimize $f_{m=1:M}(\mathbf{x}) = Dx_M + S_m h_m(x_1, ..., x_{M-1})$

Where

$$\mathbf{x} = \{x_1, ..., x_M\}$$
$$= \{\max(t_M^p, A_1)(t_1^p - 0.5) + 0.5, ..., \max(t_M^p, A_{M-1})(t_{M-1}^p - 0.5) + 0.5, t_M^p\}$$
$$\mathbf{t}^p = \{t_1^p, ..., t_M^p\} \leftarrow\!| \, \mathbf{t}^{p-1} \leftarrow\!| \cdots \leftarrow\!| \, \mathbf{t}^1 \leftarrow\!| \, \mathbf{z}_{[0,1]}$$
$$\mathbf{z}_{[0,1]} = \{z_{1,[0,1]}, ..., z_{n,[0,1]}\}$$
$$= \{z_1 / z_{1,\max}, ..., z_n / z_{n,\max}\}$$

where M is the number of objectives, $\mathbf{x}$ is a set of M underlying parameters (where x_M is an underlying distance parameter and $x_{1:M-1}$ are underlying position parameters), z is a set of $k + l = n \geq M$ working parameters (the first k and the last l working parameters are position- and distance-related parameters, respectively).

$D>0$ is a distance scaling constant, $A_{1:M-1} \in \{0,1\}$ are degeneracy constants (for each $A_i = 0$, the dimensionality of the Pareto optimal front is reduced by one), $h_{1:M}$ are shape functions, $S_{1:M}>0$ are scaling constants, and $\mathbf{t}^{1:p}$ are transition vectors, where "$\leftarrow\!|$" indicates that each transition vector is created from another vector via transformation functions. The domain of all $z_i \in \mathbf{z}$ is [0, $2i$], $i = 1, ..., n$. Note that all $x_i \in \mathbf{x}$ will have domain [0,1].

Constants $S_{m=1:M} = 2m$, $D = 1$, $A_1 = 1$, $A_{2:M-1} = 1$

11) WFG1

Shape $h_{m=1:M-1}$=convex$_m$

h_M = mixed$_M$(with α=1 and A=5)

t^1 $t^1_{i=1:k} = y_i$

$t^1_{i=k+1:n}$ = s_linear(y_i,0.35)

t^2 $t^2_{i=1:k} = y_i$

$t^2_{i=k+1:n}$ = b_flat (y_i, 0.8,0.75,0.85)

t^3 $t^3_{i=1:n}$ = b_poly(y_i,0.02)

t^4 $t^4_{i=1:M-1}$ = r_sum ($\{y_{(i-1)k/(M-1)+1}, \ldots, y_{ik/(M-1)}\}$,

$\{2((i-1)k/(M-1)+1),\ldots,2ik/(M-1)\})$

t^4_M = r_sum $(\{y_{k+1},\ldots,y_n\},\{2(k+1),\ldots,2n\}$

Optimal solutions:

$z_{i=1:k}$: any combination of values in the range [0, 2i]

$$z_{i=k+1:n} = 2i \times 0.35$$

12) WFG8

Shape $h_{m=1:M}$ =concave$_m$

t[1] $t^1_{i=1:k} = y_i$

$t^1_{i=k+1:n}$ = b_param(y_i, r_sum($\{y_1,\ldots,y_{i-1}\},\{1,\ldots,1\}$), 0.98/49.98, 0.02, 50)

t[2] $t^2_{i=1:k} = y_i$

$t^2_{i=k+1:n}$ = s_linear(y_i,0.35)

t[3] $t^3_{i=1:M-1}$ = r_sum $(\{y_{(i-1)k/(M-1)+1},\ldots,y_{ik/(M-1)}\},\{1,\ldots,1\})$

t^3_M = r_sum $(\{y_{k+1},\ldots,y_n\},\{1,\ldots,1\})$

Optimal solutions:

$z_{i=1:k}$: any combination of values in the range [0, 2i]

$$z_{i=k+1:n} = 2i \times 0.35^{(0.02+49.98(\frac{0.98}{49.98}-(1-2u)\left|\lfloor 0.5-u \rfloor+\frac{0.98}{49.98}\right|))^{-1}}$$

$$u = r\_sum\left(\{z_1,\ldots,z_{i-1}\},\{1,\ldots,1\}\right).$$

To obtain a Pareto optimal solution, the position should first be determined by setting $z_{1:k}$ appropriately. The required distance-related parameter values can then be calculated by first determining z_{k+1} (which is trivial given $z_{1:k}$ have been set), then z_{k+2}, and so on, until z_n has been calculated.

13) WFG9

Shape $h_{m=1:M}$ =concave$_m$

t[1] $t^1_{i=1:n-1}$ = b_param(y_i, r_sum($\{y_{i+1},\ldots,y_n\},\{1,\ldots,1\}$), 0.98/49.98, 0.02, 50)

$t^1_n = y_n$

t[2] $t^2_{i=1:k}$ =s_decept(y_i, 0.35, 0.001, 0.05)

$t^2_{i=k+1:n}$ =s_multi(y_i, 30, 95, 0.35)

t[3] $t^3_{i=1:M-1}$ = r_nonsep $(\{y_{(i-1)k/(M-1)+1},\ldots,y_{ik/(M-1)}\}, k/(M-1))$

t^3_M = r_nonsep $(\{y_{k+1},\ldots,y_n\}, l)$

Optimal solutions:

$$z_{i=k+1:n} = 2i \times \begin{cases} 0.35^{(0.02+1.96u)^{-1}}, & i \neq n \\ 0.35, & i = n \end{cases}$$

$$u = r\_sum\left(\{z_{i+1},\ldots,z_n\},\{1,\ldots,1\}\right).$$

Which can be found by first determining z_n, then z_{n-1}, and so on, until the required value for z_{k+1} is determined. Once the optimal values for $z_{k+1:n}$ are determined, the position-related parameters $z_{1:k}$ can be varied arbitrarily (in the range [0,2i]) to obtain different Pareto optimal solutions.

Table 1: Properties of the test functions [4]

Test functions	Objective	# Parameters	Separability	Modality	No Extremal	No Medial	Optima Known	Geometry	Pareto many-to-one	Flat Regions
1.OKA2	f_1	1	S	U	√	√	√	concave	X	X
	f_2	1	NS	M						
2.SYMPART	$f_{1:2}$	√	NS	M	√	√	√	concave	√	X
3.S_ZDT1	f_1	1	S	U	√	√	√	convex	X	X
	f_2	√	S	U						
4.S_ZDT2	f_1	1	S	U	√	√	√	concave	X	X
	f_2	√	S	U						
5.S_ZDT4	f_1	1	S	U	√	√	√	convex	X	X
	f_2	√	S	M						
6.R_ZDT4	$f_{1:2}$	√	NS	M	√	√	√	convex	X	X
7.S_ZDT6	f_1	1	S	M	√	√	√	concave	√	X
	f_2	√	S	M						
8.S_DTLZ2	$f_{1:M}$	√	S	U	√	√	√	concave	√	X
9.R_DTLZ2	$f_{1:M}$	√	NS	M	√	√	√	concave	√	X
10.S_DTLZ3	$f_{1:M}$	√	S	M	√	√	√	concave	√	X
11.WFG1	$f_{1:M}$	√	S	U	√	√	√	convex,mixed	√	√
12.WFG8	$f_{1:M}$		NS	U				concave		
13.WFG9	$f_{1:M}$		NS	M,D				concave		

S: Separable; NS: nonseparable; U: Uni-modal; M: Multi-modal; D: Deceptive.

2 Performance Assessment

Pareto Optimal Fronts: The Pareto optimal fronts of all test functions are provided in the data file.

Runs: 25

Max_FES: The maximum number of function evaluations, 5e+5 for all test functions.

The Approximation Set Size: We define the size of the resulting nondominated set to allow for a fair comparison regarding the performance metrics.

Table 2: the experimental settings for the test functions

Test Function	No. of Objectives	No.of Parameter	Approximation set size
1.OKA2	2	3	100
2.SYMPART	2	30	
3.S_ZDT1	2	30	
4.S_ZDT2	2	30	
5.S_ZDT4	2	30	
6.R_ZDT4	2	10	
7.S_ZDT6	2	30	
8.S_DTLZ2	3	30	150
	5	30	3000
9.R_DTLZ2	3	30	150
	5	30	3000
10.S_DTLZ3	3	30	150
	5	30	3000
11.WFG1	3	24*	150
	5	28*	3000
12.WFG8	3	24*	150
	5	28*	3000
13.WFG9	3	24*	150
	5	28*	3000

* The dimension of WFG test function is decided by $k+l$:
k: If M = 2, then let k = 4. Otherwise, if M >= 3, let k = 2*(M-1).
l: It is recommended that l=20 should be sufficient for the number of distance related parameters.
Therefore, for test functions (11,12,13) with 3 objectives(M=3), k = 4, l = 20, $D = k+l$ = 24;
with 5 objectives(M=5), k = 8, l = 20, $D = k+l$ = 28.

Population Size: So long Max_FES is never exceeded, you are free to employ whichever population size suits your algorithm.

Metrics:

1) **R indicator (** I_{R2} **)** [5]

$$I_{R2} = \frac{\sum_{\lambda \in \Lambda} u^*(\lambda, A) - u^*(\lambda, R)}{|\Lambda|}$$

where R is a reference set, u^* is the maximum value reached by the utility function u with weight vector λ on an approximation set A, i.e., $u^* = \max_{z \in A} u_\lambda(z)$. We choose the augmented Tchebycheff function as the utility function.

2) **Hypervolume difference to a reference set (** $I_{\bar{H}}$ **)** [5]

The hypercolume indicator I_H measures the hypervolume of that portion of the objective space that is weakly dominated by an approximation set A, and is to be maximized. Here we consider the hypervolume difference to a reference set R, and we will refer to this indicator as $I_{\bar{H}}$, which is defined as $I_{\bar{H}} = I_H(R) - I_H(A)$ where smaller values correspond to higher quality - in contrast to the original hypervolume I_H .

When calculate these two indicators, we need normalize the objective values first.

3) **Covered sets** *CS* [6]

The above 2 indicators only refer to the objective space, we therefore need one additional indicator that regard the Pareto sets for SYMPART test function.

The formal definitions refer to a population P of points (ind) in decision space and a set S of Pareto subsets (set). The Boolean function near(ind,set) becomes true if the tested individual reaches the vicinity of the tested set.

The number of covered Pareto subsets (which comprise at least one individual in their vicinity). $CS(P,S) = \left|\{\text{set} \in S : \exists \text{ind} \in P, \text{near}(\text{ind},\text{set})\}\right|$

4) **Attainment surface** [5]

An approximation set A is called the *k*%-approximation set of an EAF $\alpha_r(z)$, if it weakly dominates exactly those objective vectors that have been attained in at least *k* percent of the *r* runs. Formally, $\forall z \in Z : \alpha_r(z) \geq k/100 \Leftrightarrow A \preceq \{z\}$

where $\alpha_r(z) = \frac{1}{r}\sum_{i=1}^{r} I\left(A^i \preceq \{z\}\right)$

A^i is the *i*th approximation set (run) of the optimizer and *I*(.) is the indicator function, which evaluates to one if its argument is true and zero if its argument is false.

An ***attainment surface*** of a given approximation set A is the union of all tightest goals that are known to be attainable as a result of A. Formally, this is the set $\{z \in \mathbb{R}^n : A \preceq z \wedge \neg A \prec\prec z\}$

3 Algorithm Complexity

1) T1 $= (\sum_{i=1}^{N} t1_i)/N$. $t1_i$ = the computing time of 10000 evaluations for problem *i*. *N* is the total number of the test functions. Please consider *N*=19, counting M=3 and M=5 separately.

Note: Please make sure the T1 is the computation time of the basic function, and no additional operators are added in.

And please use loop to calculate T1:

```
for  i = 1:10000
        f = function(x)
end
```

2) *T*2 $= (\sum_{i=1}^{N} t2_i)/N$. $t2_i$ = the complete computing time for the algorithm with 10000 evaluations for problem *i*.

The complexity of the algorithm is reflected by: *T*1, *T*2, and $(T2 - T1)/T1$

4 Results Format

Participants are suggested to present their results in the following format:

PC Configuration:

System: CPU:

RAM: Language:

Algorithm:

Parameters Setting:

a) All parameters to be adjusted.

b) Corresponding dynamic ranges.

c) Guidelines on how to adjust the parameters.

d) Estimated cost of parameter tuning in terms of number of FEs.

e) Actual parameter values used.

Results Achieved:

For all the test functions, record the approximate set after 5e+3, 5e+4,5e+5 FES in each run.

For **indicator** I_{R2} and $I_{\bar{H}}$, participants are suggested to present the following: best, median, worst result, mean value and standard deviation for the 25 runs.

Table 3: The results for R indicator on test functions 1-7. (Please keep 4 digits after the decimal point as the example data in the table)

FES		1.OKA2	2.SYMPART	3.S_ZDT1	4.S_ZDT2	5.S_ZDT4	6.R_ZDT4	7.S_ZDT6
5e+3	Best	0.2356e-02						
	Median	0.5600e-01						
	Worst	0.8935e+00						
	Mean	0.4112e-01						
	Std	0.2548e-04						
5e+4	Best	…						
	Median							
	Worst							
	Mean							
	Std							
5e+5	Best							
	Median							
	Worst							
	Mean							
	Std							

Table 3(cont.): The results for R indicator on test functions 8-13 when M=3.

FES		8.S_DTLZ2	9.R_DTLZ2	10.S_DTLZ3	11.WFG1	12.WFG8	13.WFG9
5e+3	Best						
	Median						
	Worst						
	Mean						

	Std						
5e+4	Best						
	Median						
	Worst						
	Mean						
	Std						
5e+5	Best						
	Median						
	Worst						
	Mean						
	Std						

Table 3(cont.): The results for R indicator on test functions 8-13 when M=5.

…

Table 4: The results for Hypervolume indicator $I_{\bar{H}}$ on test functions 1-7.

…

Table 4(cont.): The results for Hypervolume indicator $I_{\bar{H}}$ on test functions 8-13 when M=3.

…

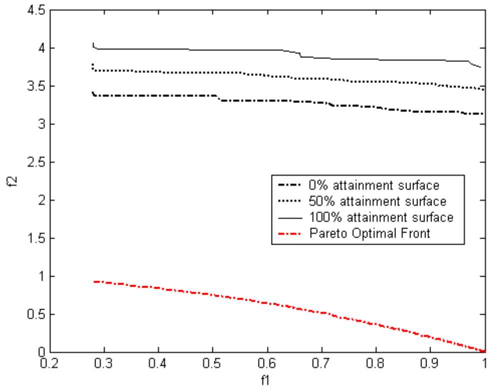
Table 4(cont.): The results for Hypervolume indicator $I_{\bar{H}}$ on test functions 8-13 when M=5.

…

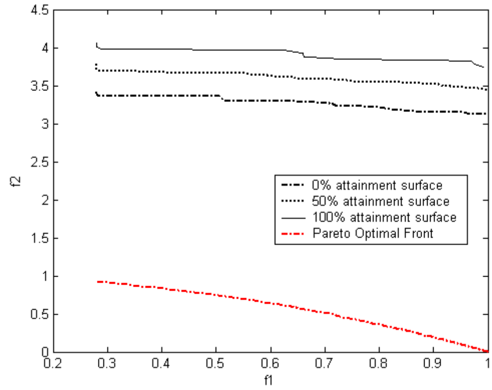
Table 5: The results for Covered sets *CS* for test function SYMPART.

FES	5e+3	5e+4	5e+5
Best			
Median			
Worst			
Mean			
Std			

Plot 0%, 50%,100% attainment surfaces after 5e+5 FES on test functions 1-7.



test function 1

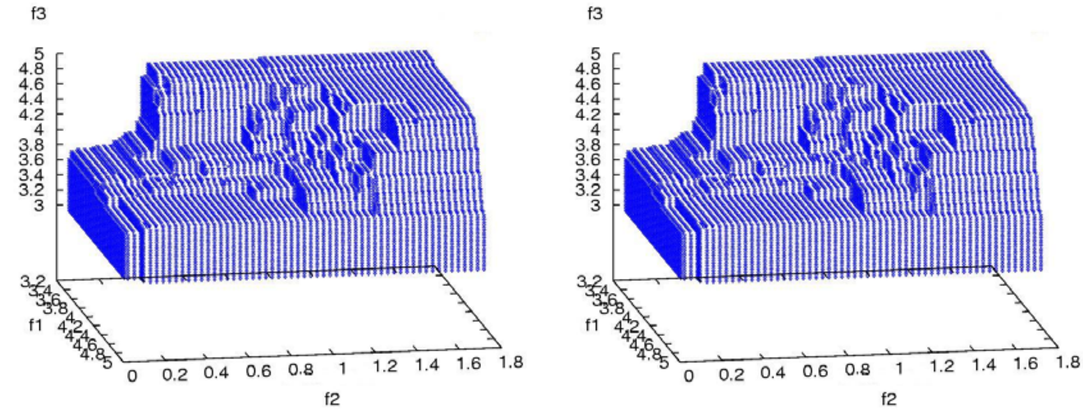


test function 2

…

Fig. 1 Pareto optimal front and 0%, 50%,100% attainment surfaces after 5e+5 FES on test function s 1-7.

Plot 50% attainment surfaces after 5e+5 FES on test functions 8-13(M=3)[7].



Test function 8 Test function 9

…

Fig. 2 50% attainment surfaces after 5e+5 FES on test function s 8-13(M=3).

Plot the Pareto front after 5e+5 on test functions 12 and 13 (M=5). (Use the median approximation set with respect to the R indicator to plot the figure.)

The figure shows the pair wise interactions among these five normalized objective functions. The axes of any plot can be obtained by looking at the corresponding diagonal boxes and their ranges. For example, the plot at the first row and third column has its vertical axis as f1 and horizontal axis as f3.

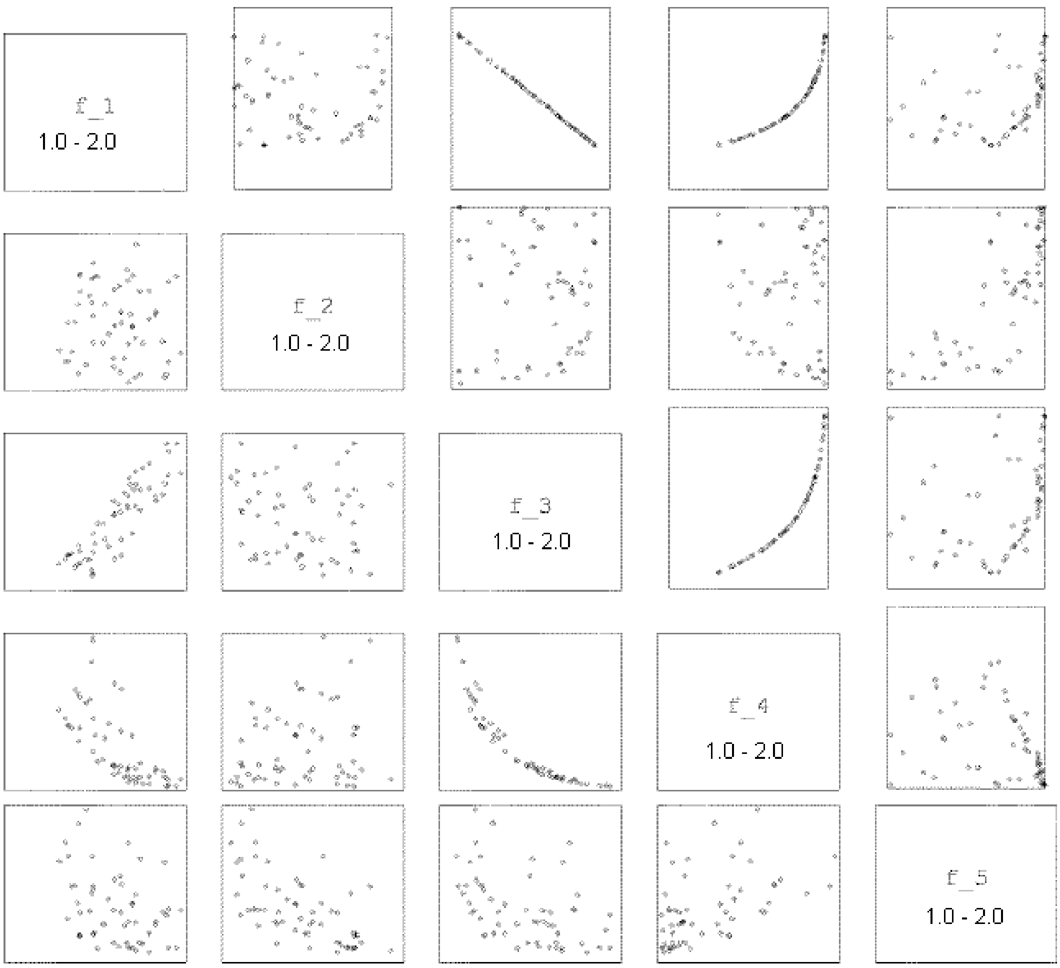


Fig. 3 Upper diagonal plots are for WFG8 (M=5) and lower diagonal plots are for WFG9(M=5).

Algorithm Complexity

T1	T2	(T2-T1)/T1

Table 4 Computational complexity

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