technische universität dortmund

Dept. of Computer Science
Chair of Algorithm Engineering

Tutorial for

Introduction to Computational Intelligence in Winter 2011/12

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http://ls11-www.cs.tu-dortmund.de/people/rudolph/teaching/lectures/CI/WS2011-12/lecture.jsp

Sheet 2, Block A

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Exercise 2.1: Clustering for RBF (5 Points)

Implement in R a k-means algorithm for clustering. Alternatively, find, download, understand and describe a public domain version.

Apply your algorithm to identify 4 clusters in the given input set.

Recall that the number of clusters shall be equal to the number of neurons in the RBF. What kind of influence has the variable σ? What happens in case of big/small values? Having determined the clusters, how shall σ be chosen? Plot the data set with the calculated cluster centers.

Exercise 2.2: Weights for RBF (5 Points)

The optimal weights $\mathbf{w}$ for an RBF net can be determined from the solution of the matrix equation $P\mathbf{w} = \mathbf{y}$ via the pseudo inverse of P.

a) Show formally that the optimal weights can be determined via minimizing
$||P\mathbf{w} - \mathbf{y}||^2 = (P\mathbf{w} - \mathbf{y})'(P\mathbf{w} - \mathbf{y}) \rightarrow min!$

Use differential calculus.

b) If the training examples lead to an ill-conditioned matrix P the numerical process can be made more stable if we minimize the objective function
$||P\mathbf{w} - \mathbf{y}||^2 + \mathbf{w}'D\mathbf{w} \rightarrow min!$,
where $D = \text{diag}(d_1, \ldots, d_q)$ is a diagonal matrix with positive diagonal entries $d_i > 0$.

Derive the expression for the optimal weights via differential calculus.

Exercise 2.3: BAM (5 Points)

Let $(x_1, y_1), \ldots, (x_m, y_m)$ be m pairs of *bipolar* row vectors that are to be stored in a bidirectional associative memory (BAM) neural network with weight matrix

$$W = \sum_{i=1}^{m} x_i' \, y_i.$$

How many pairs can be stored in this BAM under which conditions imposed on the data set?

Hint: First restrict the dimension of $x_i, y_i \in \{-1, +1\}^2$ to 2. Then try to generalize.