

Evolution and Optimum Seeking

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Preface

In 1963 two students at the Technical University of Berlin met and were soon to collaborate on experiments which used the wind tunnel of the Institute of Flow Engineering. During the search for the optimal shapes of bodies in a flow, which was then a matter of laborious intuitive experimentation, the idea was conceived of proceeding strategically. However, attempts with the coordinate and simple gradient strategies were unsuccessful. Then one of the students, Ingo Rechenberg, now Professor of Bionics and Evolutionary Engineering, hit upon the idea of trying random changes in the parameters defining the shape, following the example of natural mutations. The evolution strategy was born. A third student, Peter Bienert, joined them and started the construction of an automatic experimenter, which would work according to the simple rules of mutation and selection.

The second student, I myself, set about testing the efficiency of the new methods with the help of a Zuse Z23 computer; for there were plenty of objections to these *random strategies*. In spite of an occasional lack of financial support, the Evolutionary Engineering Group which had been formed held firmly together. Ingo Rechenberg received his doctorate in 1970 for the seminal thesis: *Optimierung technischer Systeme nach Prinzipien der biologischen Evolution*. It contains the theory of the two membered evolution strategy and a first proposal for a multimembered strategy, which in the nomenclature introduced here, is of the $(\mu+1)$ type. In the same year financial support from the Deutsche Forschungsgemeinschaft (German Research Association) enabled the initiation of the work which comprises most of the present book. This work was concluded, at least temporarily, in 1974 with the thesis *Evolutionsstrategie und numerische Optimierung* and published by Birkhäuser, Basle, Switzerland, in 1977 under the title *Numerische Optimierung von Computer-Modellen mittels der Evolutionsstrategie* as well as by Wiley, Chichester, in 1981 as monograph *Numerical optimization of computer models*.

Between 1976 and 1985 the author was not able to continue his work in the field of Evolution Strategies (nowadays abbreviated: ESs). The general interest in this type of optimum seeking algorithms was not broad enough for there to be financial support. On the other hand, the number of articles, journals, and books devoted to (mathematical) optimization has increased tremendously.

Looking back upon the development from 1964 on, when the first ES version was devoted to experimental optimization, i.e., upon 30 years, or roughly one human generation, reveals three interesting facts:

- First, ESs are not at all outdated. On the contrary, three consecutive conferences on *Parallel Problem Solving from Nature* (PPSN) in 1990 (see Schwefel and Männer, 1991), 1992 (Männer and Manderick, 1992), and 1994 (Davidor, Schwefel, and Männer, 1994) have demonstrated a revived and increasing interest.
- Secondly, the computational environment has changed over time, not only with respect to the number of (also personal) computers and their data processing power, but even more with respect to new architectures. MIMD (*Multiple Instructions*

Multiple Data) machines with many processors working in parallel for one task seem to wait for inherently parallel problem solving concepts like ESs. Biological metaphors prevail within the new branch of Artificial Intelligence, called *Artificial Life* (AL).

- Third, updating this dissertation from 1974/1975 once more (after adding only a few pages to Chapter 7 in 1981) can be done without rewriting the bulk of the chapters on traditional approaches. Since the emphasis always has been centered on derivative-free direct optimum-seeking methods, it should be sufficient to add material on three concepts now, i.e., *Genetic Algorithms* (GAs), *Simulated Annealing* (SA), and *Tabu Search* (TS). This was done with the new Sections 5.3 to 5.5 in Chapter 5.

Another innovation is a floppy disk with all those procedures which had been used for the test series in the 1970s, along with a users' manual. Hopefully, some incorrectnesses have been deleted now, too.

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