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Invited Review

Adaptive memory programming: A unified view of metaheuristics

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Abstract

The paper analyses recent developments of a number of memory-based metaheuristics such as taboo search (TS), scatter search (SS), genetic algorithms (GA) and ant colonies. It shows that the implementations of these general solving methods are increasingly similar. So, a unified presentation is proposed under the name of adaptive memory programming (AMP). A number of methods recently developed for the quadratic assignment, vehicle routing and graph colouring problems are reviewed and presented under the AMP point of view. AMP presents a number of interesting aspects such as a high parallelization potential and the ability of dealing with real and dynamic applications.

Keywords: Metaheuristics; Evolutionary computations; Genetic algorithms; Multi-agent systems; Taboo search; Quadratic assignment; Vehicle routing

1. Introduction

Generic heuristic methods, also called metaheuristics or general local search methods are growing at a very fast, exploding rate: the most important conferences in operational research have one or many sessions entirely devoted to metaheuristics and there are also journals entirely devoted to them. Among the most successful techniques, we can quote genetic algorithms (GA), simulated annealing (SA), taboo search (TS), scatter search (SS) and ant systems (AS). The success of these methods depends on many factors, like their ease of implementation, their ability to consider specific constraints that arise in practical applications and the high quality of the solutions they produce.

From a theoretical point of view, however, the use of these methods has not yet been justified. For example, a few convergence theorems for SA or TS exist (Aarts and Van Laarhoven, 1985; Hajek, 1988; Faigle and Kern, 1992) but they are useless in practice. These theorems simply state that the

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search has a very high probability of ending with an optimal solution if a disproportionate computing time is allowed (larger, in fact, than the time needed for a complete enumeration of the solution space (Aarts and Van Laarhoven, 1985)). But these metaheuristics are really competitive in practice. In this race for competitiveness, the most efficient methods hybridize two or more metaheuristics. The result is that similar problem-solving methods are known under very different names like genetic hybrids, probabilistic TS, adaptive multi-start or MAX–MIN ant system. All these methods have three features in common: first, they memorize solutions or characteristics of solutions generated during the search process; second, they include a procedure that creates an initial solution with the information stored in memory; third, they apply a local search method, like a local search improvement method, an elementary TS or SA to improve the initial solution.

Observing the similarities between these methods, a unified presentation is useful and necessary, as well as a more synthetic and general name. The proposed terminology for grouping these metaheuristics under the same roof is adaptive memory programming (AMP). This term was already proposed by Glover in connection with taboo search (Glover, 1997; Glover and Laguna, 1997). It is true that TS has always been presented as an open technique that can include components from various fields, in particular artificial intelligence. The inclusion of a memory and a learning process within the TS framework was thus proposed very early: even the name of the method comes from its short term memory component! However, the evolution of a particular TS implementation can very well produce a method without any taboo list (for example, if parameter tuning indicates that a taboo list size of 0 is best). So, the name of the method becomes hard to justify, but for historical reasons.

The same phenomenon is observed with other metaheuristics. For example, most efficient implementations of GA do not encode solutions of the problem as binary vectors. Furthermore, the cross-over and mutation operators have often little or nothing to do with the standard operators for which some theoretical results have been obtained.

The aim of this paper is thus to propose the name AMP which is more general and seems to better reflect the current reality. To justify the use of this term, we first review in Section 2 various metaheuristics that can be viewed as AMP methods, at least for their most recent and efficient implementations. Section 3 then presents our views on AMP and shows that many applications for different combinatorial optimization problems perfectly fit in this framework.

2. Metaheuristics with memory

The term *memory* was used explicitly for TS only, but a number of other metaheuristics use mechanisms that can be considered as memories. For GA and SS, the memory is constituted by a population of solutions; for AS, the pheromone trail is also a form of memory. So, let us first review a number of memory-based metaheuristics.

2.1. Genetic algorithms

Basically, GA simulate the evolutionary process of species that sexually reproduce. This evolutionary process can be described as follows. During sexual reproduction, a new individual, different from its two parents, is created through the action of two fundamental mechanisms. The first one is cross-over which combines half of the genetic patrimony of each parent to produce the genetic patrimony of the new individual. The second one is mutation by which a spontaneous modification of the genetic patrimony occurs. The new individual so created (child or offspring) will therefore be different from its parents, but will also share a number of their characteristics. If the child inherits good characteristics from its parents, its survival probability will be higher, as compared to individuals that inherit bad characteristics. It will thus have a higher probability to reproduce and disseminate good characteristics to its offspring.

The analogy between this evolutionary process and the GA metaheuristic initiated by Holland

(1975) (who was not so much interested in optimization) can be established in the following way. An individual is associated with a feasible solution of the problem at hand. The solution is encoded as a binary vector. The cross-over operator then exchanges sub-strings taken from both parents to produce an offspring. Mutation is a secondary operator that flips bit values on the offspring vector, with a small probability at each position. The quality of the newly created vector is finally evaluated according to the objective value of the solution it encodes.

A simple genetic algorithm can be sketched as follows:

1. Generate a population of vectors (individuals).
2. While a stopping criterion is not met do:
   2.1. select, with replacement, a pool of parent vectors from the population;
   2.2. randomly match the parent vectors and apply cross-over to produce offspring vectors;
   2.3. apply mutation to each offspring;
   2.4. evaluate offspring;
   2.5. insert offspring in the population;
   2.6. eventually, remove individuals from the population with a culling operator.

Typically, this procedure stops after a fixed number of iterations (generations) or when the population does not improve any more. It is worth noting that the selection of parents is probabilistically biased towards the best individuals. Hence, the latter are more likely to disseminate their good characteristics to offspring (like the corresponding natural evolution process). Recent implementations of GA do not always use a binary vector for coding a solution but they use a representation that is better adapted to the application domain. For example, permutation of integer values are widely used for the travelling salesman problem (TSP) or the quadratic assignment problem (QAP). This influences directly the cross-over and mutation operators. While the coding scheme was the key point of earlier genetic implementations, the most important elements of recent implementations are the definition of cross-over and mutation operators that are able to extract relevant (and a priori unknown) information on the structure of good solutions.

2.2. Scatter search

SS has been proposed by Glover (1977) to solve integer programming problems. The method is rather similar to GA, at least if one considers the most recent ways of implementing the genetic metaphor. The SS method is based on a population of solutions (integer vectors) that evolves through selection, linear combination, integer vector transformation and culling to produce a new population of solutions. With respect to a standard genetic algorithm, SS has the following particularities:

- binary vectors are replaced by integer vectors;
- more than two parents can be matched to produce a new vector;
- cross-over is replaced by a convex or non-convex linear combination of two or more vectors;
- mutation is replaced by a procedure that repairs or projects the newly created vector in the space of feasible solutions.

These particularities can also be seen as generalizations of the basic GA procedure. In fact, such generalizations have later been proposed and exploited by various authors (see for example, Potvin and Bengio, 1996; Mühlenbein et al., 1988). Let us quote:

- departure from the binary vector scheme;
- use of a variable number of parents to produce an offspring;
- development of specialized cross-over operators;
- use of local search methods to improve solutions obtained through cross-over;
- use of repair operators;
- the solutions that are kept in the population from one iteration to the next are chosen with the help of an elaborated clustering method rather than a simple culling operator.

2.3. Taboo search

Among the metaheuristics with memory presented in this section, TS is the only one that has been explicitly developed with a memory, or more exactly, with a set of memories. The term TS has been proposed for the first time by Glover (1986). The basic idea of this method is to locally and

repeatedly modify a solution while memorizing these modifications to avoid visiting the same solutions twice or in a cyclic manner. To this end, modifications or characteristics of modifications are stored in lists that forbid their use for a certain number of iterations, thus leading to the names of taboo list and TS.

In a sense, this method imitates a human being looking for a good solution of a combinatorial optimization problem. First, an initial solution is looked for, even a bad one. Then, this solution is iteratively improved through local modifications. The latter modifications do not necessarily improve the solution at each iteration but the aim is to direct the search toward a good subset of solutions.

Later, Glover (1989, 1990) proposed a number of strategies to guide the search and make it more efficient; at the same time, he emphasized that TS was open to any strategy well adapted to the problem on which it is applied. In this sense, the metaheuristics with memory presented in this section are implicitly contained in TS. We think, however, that the term TS does not appropriately characterize their most recent implementations.

The most important idea of the first publication on taboo search (Glover, 1986) is the use of a short term memory, known as the taboo list. Numerous applications, even recent ones, still only exploits this mechanism. The use of long term memories spreads well after the "complete" description of TS by Glover (1989, 1990). These are found under different forms, from statistics on modifications made to the current solution (Taillard, 1991, 1993, 1994; Soriano and Gendreau, 1996) to complete solutions of high quality visited during the search. Their exploitation for guiding the search can also vary from a perturbation of the objective function value to the construction of a brand new starting solution.

A basic TS can be sketched as follows:

1. Generate an initial solution s_0; initialize the memories; $k \leftarrow 0$; $s^* \leftarrow s_0$.
2. While a stopping criterion is not met do:
   2.1. choose s_{k+1}, a neighbour solution of s_k, using data stored in the memories;
   2.2. if s_{k+1} is better than s^* then $s^* \leftarrow s_{k+1}$;
   2.3. $k \leftarrow k + 1$;
   2.4. update the memories.

Typically, the stopping criterion corresponds to a fixed number of iterations or a number of consecutive iterations without improving the best known solution s^*. Each step of the above algorithm can be very simple or very complicated. For example, the choice of s_{k+1} in step 2.1 may imply that all neighbours of s_k are examined and the best non-taboo solution is chosen; it may also imply the construction, in a complex way, of a solution that is not so close of s_k, through another embedded metaheuristic, as in Rochat and Taillard (1995).

However, TS is composed of various other principles. Among the most important ones, let us quote strategic oscillations, based on alternating intensification and diversification phases. These phases are often implemented by repeatedly building new solutions before running a basic TS for a given number of iterations. The new solutions built can be similar to the best solutions found by the search so far (intensification) or different from the solutions visited (diversification).

2.4. Ant systems

The idea of imitating the behaviour of ants to find solutions to combinatorial optimization problems was initiated by Colorni et al. (1992a,b). The metaphor comes from the way ants search for food and find their way back to the nest. Initially, ants explore the area surrounding their nest in a random manner. As soon as an ant finds a source of food, it evaluates the interest of the source (quantity and quality) and carries some of the food to the nest. During the return trip, the ant leaves on the ground a chemical pheromone trail whose quantity depends on the quality of the source. The rôle of this pheromone trail is to guide other ants toward the source. After a while, the path to a good source of food will be indicated by a large pheromone trail, as the trail grows with the number of ants that reach the source. Since sources that are close to the nest are visited more frequently than those that are far away, pheromone trails leading to the nearest sources grow faster. The final result of this process is that ants are able to optimize their work.

The transposition of this food searching behaviour into an algorithmic framework for solving combinatorial optimization problems is obtained through an analogy between:

- the search area of the real ants and the set of feasible solutions to the combinatorial problem;
- the amount of food associated with a source and the objective function;
- the pheromone trail and an adaptive memory.

A detailed description of how such analogies can be applied in the case of the TSP may be found in Dorigo et al. (1996). A standard ant system is schematically described as follows:

1. Initialize the pheromone trail.
2. While a stopping criterion is not met do:
   2.1. for each ant, construct a new solution using the current pheromone trail and an evaluation of the partial solution being constructed;
   2.2. update the pheromone trail.

The most important component of an ant system is the management of pheromone trails. In a standard ant system, pheromone trails are used in conjunction with the objective function to guide the construction of new solutions. Once a solution has been produced, a standard ant system updates the pheromone trails as follows: first all trails are weakened to simulate the evaporation of pheromone; then, pheromone trails that correspond to components that were used to construct the resulting solution are reinforced, taking into consideration the quality of this solution.

Based on the previous general scheme different AS implementations have been proposed where pheromone updating is performed in different ways: in Colorni et al. (1992a) the pheromone is updated by all ants involved in the optimization process while in Gambardella and Dorigo (1996) only the best ant is allowed to update pheromone information. MAX–MIN ant system (Stützle and Hoos, 1999) introduces an updating phase constrained between threshold values. Different ways of modifying pheromone values generate different types of search mechanism: for example, in Gambardella and Dorigo (1996) the result of the updating phase is to drive the search around the neighbourhood of the best solutions found so far. All the mentioned AS have been applied to different combinatorial optimization problems like symmetric and asymmetric TSPs (Colorni et al., 1992a; Gambardella and Dorigo, 1996; Dorigo et al., 1996) and quadratic assignment problems (Dorigo et al., 1996) with comparable or even better performances than other natural inspired systems. More recently it has been shown that AS based algorithms are very powerful in combination with local search procedures. In these situations pheromone information is used to produce solutions (diversification phase) that are optimized by a local search (intensification phase). Optimized solutions are then used to update pheromone information and new solutions are successively generated by the ants. In particular let us mention successful hybrid ant system implementations applied to symmetric and asymmetric TSPs (Gambardella and Dorigo, 1996; Dorigo and Gambardella, 1997; Stützle and Hoos, 1999), quadratic assignment problems (Taillard, 1998; Gambardella et al., 1999; Stützle and Hoos, 1999) and vehicle routing problems (Bullnheimer et al., 1999; Gambardella and Dorigo).

3. Adaptive memory programming

Although the methods presented in the previous section may look very different at first glance, an analysis of a number of implementations, and especially the most efficient and recent ones, shows that they are sometimes extremely close in their working principles. In other terms, one observes a unification of different metaheuristics in such a way that it now becomes difficult to make clear distinctions. Indeed, the population of a genetic algorithm or SS, or the pheromone trail of an ant system, can be considered as a special kind of memory, like the memories found in TS. Conversely, a TS that periodically restarts from new solutions constructed from a memory (a population of solutions and/or a frequency-based memory) can be assimilated to a genetic algorithm or an ant system. Hence, a large number of efficient methods for solving combinatorial optimization problems now share common characteristics. More precisely:

1. a set of solutions or a special data structure that aggregates the particularities of the solutions produced by the search is memorized;
2. a provisional solution is constructed using the data in memory;
3. the provisional solution is improved using a local search algorithm or a more sophisticated metaheuristic;
4. the new solution is included in the memory or is used to update the data structure that memorizes the search history.

These recent methods are thus characterized by the exploitation of a memory to construct a new solution, an improvement procedure to find an even better solution and, finally, a memory update procedure based on pieces of knowledge brought by the improved solution. It thus looks appropriate and natural to speak of AMP for this type of metaheuristics. From an algorithmic point of view, an AMP may be sketched as follows:

1. Initialize the memory.
2. While a stopping criterion is not met do:
   2.1. Generate a new provisional solution s using data stored in the memory.
   2.2. Improve s by a local search; let s' be the improved solution.
   2.3. Update the memory using the pieces of knowledge brought by s'.

There are other metaheuristics that share a number of characteristics with this AMP scheme. Let us mention the memetic algorithms of Moscato (1999), the greedy randomized adaptive search (GRASP) of Feo and Resende (1995), the adaptive multi start (AMS) of Boese et al. (1994) and the memory adaptive reasoning of Patterson et al. (1999). There are also metaheuristics that cannot enter the AMP scheme, such as the well known SA and threshold accepting. Indeed, the basic versions of these metaheuristics work without memory. However, they may be included in the improvement procedure of an AMP.

In this section, we review a number of recent and highly efficient applications, that can be naturally presented as AMP methods, for the quadratic assignment (QAP), vehicle routing (VRP) and graph colouring problems. We briefly sketch all these methods, emphasizing how the four main characteristics of AMP have been used, namely: the memory, the construction of a provisional solution, the improvement procedure and the memory update procedure.

3.1. Quadratic assignment problem

In the QAP, a number of units must be assigned to the same number of locations. Given the distance between each pair of locations and a flow between each pair of units, an assignment of units to locations (i.e., a permutation) is searched for that minimizes the sum of the distance $\times$ flow products over the assigned unit–location pairs. Therefore, a QAP solution can be viewed as a permutation. Many different AMP methods have been designed for the QAP. They are among the most efficient ones, in particular for real applications, where the flow and distance coefficients exhibit a very high variance. These methods are:

- hybrid TS-genetic algorithm of Fleurent and Ferland (1996);
- SS of Cung et al. (1997);
- hybrid ant system of Gambardella et al. (1999);
- fast ant system of Taillard (1998).

They are reviewed in the following.

3.1.1. Genetic hybrid of Fleurent and Ferland

3.1.1.1. Memory. The memory is a population made of a number of best solutions found by the search.

3.1.1.2. Provisional solution. The provisional solution is built using a cross-over operator designed by Tate and Smith (1995). First, two solutions (permutations) are selected from the population. The elements of the provisional permutation are then chosen in three phases:

1. elements common to both solutions are copied in the new permutation at the same position;
2. if possible, the other elements are randomly chosen from the selected solutions and copied in the new permutation at the same position;
3. the unfilled positions are randomly chosen to complete the permutation.

3.1.1.3. Improvement procedure. The provisional solution is improved by means of a short TS due to Taillard (1991).

3.1.1.4. Memory update. The new solution is inserted in the memory and the worst solution is removed from it.

This application illustrates a typical evolution of GA: the binary coding scheme is replaced by a natural representation of the solutions, the crossover operator is replaced by a relatively elaborated procedure and the mutation operator is replaced by an elaborated local search. In Taillard (1998), it is shown that the use of a simpler and faster local search for improving the provisional solutions may be more efficient than the use of TS (see also Table 2).

3.1.2. Scatter search of Cung et al. (1997)

3.1.2.1. Memory. The memory stores a number of best solutions.

3.1.2.2. Provisional solution. A number of solutions are selected in a probabilistic way from the memory. The choice of the solutions to be combined is done according to a policy that ensures a certain level of diversity in the memory. The selected solutions are manipulated as vectors of integers and a linear combination of these vectors is computed: the provisional solution is then the permutation which is closest from the previously computed vector. Cung et al. (1997) have shown that this closest permutation can be found by solving a linear assignment problem. Instead of solving this assignment problem exactly, Cung et al. (1997) use a faster heuristic method.

3.1.2.3. Improvement procedure. The provisional solution is improved by means of a basic TS. The number of iterations performed by this taboo is modulated from one call to the other.

3.1.2.4. Memory update. The new solution is inserted in the memory and the worst solution is removed from it.

3.1.3. Fast ant system of Taillard

3.1.3.1. Memory. The memory is a square matrix that records frequency statistics on the position of each element in the permutations (solutions) produced by the system.

3.1.3.2. Provisional solution. A new permutation is sequentially constructed in a probabilistic way. The probability of inserting a given element at a given position is proportional to its frequency value stored in memory.

3.1.3.3. Improvement procedure. The solution is improved by means of a very fast local search that does not necessarily reach a local optimum.

3.1.3.4. Memory update. The frequencies are recomputed at each iteration, considering both the solution just produced and the best solution ever produced by the search. The memory is cancelled and re-initialized each time the best solution known is improved in order to intensify the search. A diversification mechanism is also implemented in case the provisional solution generated is the same as the best solution produced so far.

The MAX–MIN ant system of Stützle and Hoos (1999) is similar to this method but uses slightly different statistics as memory. The improvement procedure performs a complete local search reaching the first local optimum and the diversification/intensification mechanism is different: the values contained in the statistics-matrix are lower and upper bounded by two parameters, hence the name of the method.

3.1.4. Hybrid ant system of Gambardella et al. (1999)

3.1.4.1. Memory. The memory is divided in two parts: the first part contains a matrix that records frequency statistics on solutions produced so far by the search; the second part contains a small population of solutions.

3.1.4.2. Provisional solution. As opposed to basic AS where a new solution is constructed from scratch, this method modifies the solutions found

in the population in the spirit of a local search, but using the frequency matrix to choose the modifications to perform.

3.1.4.3. Improvement procedure. Each solution in the population is improved by means of a fast local search that does not necessarily reach a local optimum.

3.1.4.4. Memory update. The memories are updated as follows. First, each solution in the current population is replaced by the solution obtained after the improvement procedure, unless the latter is worser than the original solution. Second, the frequency matrix is updated: it records a moving average of the frequency (weighted by solution quality) at which a given unit is found at a given position in the best solution of the current population.

A number of these AMPs are compared in Taillard (1995), Stützle and Hoos (1999), Gambardella et al. (1999), Taillard (1998) and the main conclusions are the following. All AMPs are performing well for instances that exhibit a high variance in the entries of the data matrices. Such instances are typically arising in real applications and often present a large "flow dominance" criterion (Vollmann and Buffa, 1966) or a large "ruggedness" value (Angel and Zissimopoulos, 1998). To obtain the best solutions in the shortest time, the fast ant system of Taillard (1998) is one of the best methods. If solutions of higher quality are desired, it seems that keeping a number of solutions in memory can be recommended. Implementations of simulated annealing, GA or variable neighbourhood search (VNS) that do not exploit either memories or local search are less efficient than AMPs (Taillard, 1998).

Table 1 compares the performances of various methods running for a short time (equivalent to 20 calls to the improvement procedure embedded in the fast ant system). The methods compared are: fast ant system (FANT, Taillard, 1998), a basic TS (Taillard, 1991) (without intensification or diversification), VNS, reduced variable neighbourhood search (RVNS) and SA (Connolly, 1990). We have (re-)implemented all these methods and run them on the same computer. All the methods were run for the same computational time, corresponding to 20 FANT iterations, $26n$ TS iterations, 20 VNS iterations, $10n^2$ RVNS iterations and $40n^2$ SA iterations (where n is the size of the problem). We have considered a selection of instances from the QAPLIB. These instances originate from real life or have been randomly generated according to distribution laws that are similar to those observed in real problems. The flow dominance of these instances is high but for the instances nug.. and sko.. The quality of the methods is measured in percentage above the best solution known, averaged over 10 runs of each algorithm. In this table, we see that FANT is generally competitive for the instances with high flow dominance. It is less competitive than TS and SA for the relatively uniform instances nug.. and sko.. In this table, we have not considered other AMPs, since they require a larger computational effort (initialization of the memory).

The efficiency (or inefficiency) of AMP can be tentatively explained as follows: If the problem has a strong structure (like els19 or tai..b instances), the memory is able to efficiently direct the search toward solutions with good general properties. Then, the improvement procedure directs the search toward very good solutions. For such instances, methods like basic taboo searches can be trapped in very bad local optima. Conversely, if the problem has no structure, the memory is not able to perform a self-organization and the AMP fails to find good solutions.

Table 2 compares the performances of various AMPs for longer runs. The methods compared are fast ant system (FANT, Taillard, 1998), genetic-descent hybrid (GDH, a modified version of the method of Fleurent and Ferland (1996) in which the TS has been replaced by the same fast improvement procedure as used in FANT, see Taillard, 1998), hybrid ant system (HAS–QAP, Gambardella et al., 1999) and TS-genetic hybrid (TSGH, Fleurent and Ferland, 1996). The computational time is equivalent to 1000 calls to the improvement procedure embedded in FANT, HAS–QAP or GDH and 130 calls to the TS embedded in TSGH. We can see in this table that the AMPs are generally finding very good solutions but for problems with low flow dominance. FANT

Table 1
Comparison of various methods for short runs, quadratic assignment instances

Problem		Quality (% above best solution known)					Time (s)
Name, size	Dominance	FANT	TS	VNS	RVNS	SA	Sun Sparc 5
bur26a	275	0.11	0.21	0.21	0.29	0.21	0.92
bur26b	275	0.16	0.30	0.24	0.44	0.34	0.94
bur26c	228	0.09	0.22	0.22	0.46	0.14	0.93
bur26d	228	0.01	0.26	0.27	0.19	0.47	0.91
bur26e	254	0.01	0.21	0.16	0.27	0.20	0.91
bur26f	254	0.01	0.35	0.19	0.27	0.36	0.91
bur26g	280	0.02	0.28	0.16	0.54	0.18	0.92
bur26h	280	0.01	0.41	0.19	0.35	0.49	0.90
els19	531	3.08	22.27	14.77	13.90	24.36	0.32
kra30a	150	3.26	3.94	5.34	7.03	3.83	1.39
kra30b	150	2.99	1.74	4.52	4.76	1.73	1.37
nug20	104	1.27	0.96	3.52	3.93	0.82	0.37
nug30	113	1.64	0.61	2.67	3.67	0.86	1.43
sko42	109	1.66	1.04	2.28	3.32	1.20	4.55
sko49	109	1.66	0.80	1.93	2.97	0.90	7.61
sko56	111	1.65	0.89	1.98	2.80	0.91	11.88
sko64	108	1.59	0.82	2.24	2.48	0.68	18.11
sko72	107	1.53	1.04	1.97	2.27	0.71	26.59
sko81	107	1.49	0.71	1.33	1.94	0.72	38.33
sko90	108	1.45	0.72	1.52	2.15	0.86	54.20
tai20b	333	0.81	17.09	9.49	6.30	13.38	0.41
tai25b	310	1.19	14.13	6.25	10.59	18.74	0.81
tai30b	324	1.74	14.09	7.56	12.92	13.85	1.55
tai35b	310	0.87	10.61	4.62	5.29	10.81	2.56
tai40b	317	2.66	9.22	4.86	9.69	7.18	4.06
tai50b	314	1.49	7.90	4.52	5.11	4.61	8.47
tai60b	318	1.12	10.21	2.72	5.91	8.58	15.47
tai80b	323	2.54	5.03	3.19	4.98	5.68	38.33
Average		1.29	4.50	3.18	4.10	4.39	

is slightly worse than HAS–QAP and GDH, indicating that the use of a richer memory that contains a number of solutions seems recommended for long runs. TSGH is worse than GDH due to the use of a slower improvement procedure.

3.2. Vehicle routing problem

The aim of the vehicle routing problem (VRP) is to determine optimal collection or delivery tours for transportation vehicles through a set of customer locations, subject to various constraints. Typically, "optimal" refers to a solution of minimum length or a solution that uses the minimum number of vehicles (when this number is not fixed).

Many different VRPs have been successfully addressed in the past with either TS or GA. For example, TS has been applied to the capacitated VRP (CVRP), where a single type of constraint states that the total load on a tour cannot exceed vehicle capacity (Osman, 1993; Taillard, 1993; Gendreau et al., 1994; Xu and Kelly, 1996), the VRP with time windows (VRPTW), where customers must be serviced within specified time intervals (in addition to the capacity constraint) (Potvin et al., 1996; Chiang and Russell, 1997), and the VRP with back hauls and time windows (VRPBTW) where customers can be either pick-up

Table 2
Comparison of various AMPs for long runs, quadratic assignment instances

Problem	Quality (% above best solution known)				Time (s)
Name	FANT	HAS–QAP	GDH	TSGH	Sparc 5
bur26a	0.03	0	0.03	0.02	46
bur26b	0.02	0	0.04	0.03	45
bur26c	0	0	0	0	46
bur26d	0	0	0	0	45
bur26e	0	0	0	0	46
bur26f	0	0	0	0	45
bur26g	0	0	0	0	46
bur26h	0	0	0	0	45
els19	0	0	1.44	0	31
kra30a	1.03	0.63	1.08	0.36	74
kra30b	0.09	0.07	0.17	0.06	83
nug20	0.14	0	0.02	0	22
nug30	0.25	0.10	0.16	0.04	78
sko42	0.24	0.08	0.09	0.19	237
sko49	0.24	0.14	0.19	0.20	400
sko56	0.32	0.10	0.15	0.30	619
sko64	0.19	0.13	0.14	0.38	951
sko72	0.35	0.28	0.20	0.39	1382
sko81	0.26	0.14	0.17	0.43	2018
sko90	0.40	0.23	0.19	0.43	2809
tai20b	0.09	0.09	0.09	0	24
tai25b	0	0	0.01	0.06	48
tai30b	0	0	0.03	0.14	85
tai35b	0.04	0.03	0.02	0.26	143
tai40b	0.20	0	0.20	0.74	228
tai50b	0.21	0.19	0.01	0.64	466
tai60b	0.25	0.05	0.01	0.83	830
tai80b	0.82	0.67	0.22	1.74	2041
Average	0.18	0.10	0.17	0.26	

or delivery points (Duhamel et al., 1997). Applications of GA for variants of the VRPTW may also be found in Blanton and Wainwright (1993), Potvin and Bengio (1996), Thangiah et al. (1991, 1993), Thangiah (1995). These implementations, mostly realized before 1995, can hardly be qualified as AMP methods, with the exception of Potvin and Bengio (1996) where a genetic algorithm is hybridized with a local search heuristic, in the same spirit as the genetic hybrid of Fleurent and Ferland (1996) for the QAP. In 1994, however, Taillard (1994) developed a true AMP method, where a TS heuristic is used to improve provisional solutions constructed from an adaptive memory. The method was then tested on the VRPTW and good results were published in Rochat and Taillard (1995). Later, Taillard et al. (1997) designed another AMP method for a variant of the VRPTW, known as the VRP with soft time windows (VRPSTW), where customer locations can be visited outside of their associated time window (at the expense of a penalty in the objective value). A parallel version of the latter algorithm was implemented on a network of workstations (Badeau et al., 1997) and adapted for a real-time version of the problem, where customer requests occur in an on-going fashion (Gendreau et al., 1996, 1999). These articles report numerical results obtained with various methods: Two AMPs that use either a simple descent method or a TS as improvement

procedure, and four different implementations of insertion and rebuild methods that are using no adaptive memory. It is shown that memory-based heuristics are clearly superior to the other methods.

Adaptations of the original AMP method of Taillard have also been applied to other variants of the VRP, namely: the VRP with multiple uses of vehicles (VRPM), where each vehicle is allowed to perform many tours during a work day (Taillard et al., 1996); the min–max CVRP and min–max multiple TSP (MTSP) where the goal is to minimize the length of the longest tour (Golden et al., 1997); the VRP with heterogeneous, or non-identical, vehicles (VRPHE) (Taillard, 1999); and two special cases of the latter problem, the vehicle fleet mix (VFM) and vehicle fleet mix with variable routing cost (VFMVRC) problems, where a fixed cost is incurred for using a given vehicle type, as well as a different cost by unit of length travelled by each vehicle type.

For all these variants, efficient methods were designed using the AMP framework. The basic scheme is the following:

3.2.1. Memory

The memory contains a set T of tours belonging to solutions produced by the search.

3.2.2. Provisional solution

A partial solution s is constructed by selecting a number of tours contained in the memory, in a probabilistic manner (i.e., tours associated with better solutions have a higher probability of being selected). More precisely, the provisional solution is built as follows:

1. Make a copy T' of T: $T' \leftarrow T$.
2. Set $s = \emptyset$.
3. While $T' \neq \emptyset$ repeat:
   3.1. Randomly choose a tour $t \in T'$.
   3.2. Set $s \leftarrow s \cup \{t\}$.
   3.3. $\forall t' \in T'$ such that $t' \cap t \neq \emptyset$, set $T' \leftarrow T' \setminus \{t'\}$.

In this procedure, Step 3.3 looks for tours that share one or more customers with the current selected tour t. These tours are removed and are not considered in the remainder of the selection process. Hence, the provisional solution obtained at the end is necessarily made of mutually exclusive tours. However, it is possible that a number of customers remains not serviced. These customers are typically introduced in the provisional solution through some least-cost insertion method to produce a complete solution. The insertion may also be performed by means of the improvement procedure if the latter is able to make partial solutions complete.

3.2.3. Improvement procedure

The provisional solution s is improved using TS.

3.2.4. Memory update

The tours of the improved solution are inserted in the memory T and the tours of the worst solution are removed from T.

For a few variants of the VRP, the AMP solves simpler auxiliary problems. A solution of the original problem is then found by (implicitly) enumerating all solutions that can be constructed with tours contained in the memory, so that all constraints of the original problem are satisfied.

The AMPs described above are able to produce solution of very good quality, especially for irregular problems (i.e., problems for which the customers are not uniformly spread on the plane and/or the quantities ordered by the customers exhibit a very high variance). So, the conclusions derived for the performance of AMP applied to the quadratic assignment problem seem also valid for VRPs. The reader is referred to the articles mentioned above for detail numerical results.

3.3. Graph colouring

Graph colouring is aimed at partitioning a set of vertices into a given number of subsets (colours) such that the number of edges that connect two vertices belonging to the same subset is minimized. In Zufferey (1998), the idea is to use a memory and a procedure for producing provisional solutions similar to that presented just above for the VRP. Indeed, instead of storing tours in T, i.e., subsets of customers, it is possible to store subsets of vertices having received the same colour in a

solution. Then, the procedure presented above for the VRP can be applied to build a partial colouring that is improved by means of a TS (Hertz and de Werra, 1987). So, the method goes along the lines of the AMP of Rochat and Taillard (1995).

Costa et al. (1995) and Fleurent and Ferland (1996) have proposed particularly effective AMP methods for graph colouring that are designed in the spirit of a genetic algorithm in which a local search is embedded. They both use the same memory and memory update mechanisms, but differ in the way they construct and improve the provisional solution. We now briefly sketch these two methods.

3.3.1. Memory

The memory stores a number of solutions.

3.3.2. Provisional solution

Two solutions are selected from the memory and combined to obtain the provisional solution. Then, the colour of each vertex of the provisional solution is copied from the selected solutions. This copy is performed according to a policy that slightly differs in both methods.

3.3.3. Improvement procedure

Costa et al. (1995) use a simple local search for improving the provisional solution while Fleurent and Ferland use a relatively elaborated TS (Hertz and de Werra, 1987).

3.3.4. Memory update

Both methods insert the new solution in the memory and remove the worst one from it.

3.4. Synthesis

Even if the list of successful applications of AMP methods to solve combinatorial optimization problems, as found in Sections 3.1–3.3, is far from exhaustive, it should clearly demonstrate that all these methods use the same working principles. We must however, emphasize that they differ a lot in their implementation details; these details should not be overlooked, as they are responsible for the success or failure of a given AMP method. In fact, there is almost an infinite number of ways of implementing a simple local search, a TS, a genetic algorithm, etc.

While promoters of different metaheuristics with memory developed increasingly sophisticated mechanisms to maintain the competitiveness of their approach, their efforts have often converged to the same general principles: use of a memory and procedures for constructing a new solution and for improving it. A notable exception is simulated annealing, as it does not use any memory. The framework of this technique being more rigid, as compared to other metaheuristics, it may explain why it did not evolve in the same way.

Why did metaheuristics with memory converge to the same basic approach? One answer could be that the a natural way of alleviating the weaknesses of a given technique is to borrow components from another technique (which does not exhibit the same weaknesses). Shortly, this evolution could be described as follows for the different approaches.

3.4.1. Genetic algorithms

Coding a solution with a binary vector is not natural and can significantly impact the performance. Hence, binary coding was replaced by a more natural representation of solutions.

The classical cross-over operators do not correspond to logical operations on solutions. Furthermore, the use of other representations than binary vectors naturally led to the design of specialized operators, well adapted to the solution representation and capable of generating new feasible solutions.

GA can easily identify different solution subspaces with good characteristics, but they lack the "killer instinct" that would allow them to intensify the search in these areas. To alleviate this weakness, the mutation operator was replaced by repair procedures and local search.

3.4.2. Scatter search

This technique initially contained all the ingredients of an AMP. This technique was not exploited for a long time but has recently emerged when its similarity with other modern techniques

was noticed. Modern forms of SS incorporate the learning capabilities provided by TS.

3.4.3. Ant system

Like GA, early implementations of ant system converged too slowly toward high quality solutions. Therefore, intensification mechanisms were gradually introduced. For example, only the best ant is allowed to update pheromone information in Gambardella and Dorigo (1996). The most recent implementations incorporate local search mechanisms to improve the solutions produced by the ants.

3.4.4. Taboo search

TS also contained all elements of the AMP framework, at least in its complete version in Glover (1989, 1990). However, a number of implementations, even recent ones, use only the basic ingredients found in Glover (1986). The main advantage of the basic version is its aggressiveness: the search converges toward a local optimum and examines the neighbourhood of this local optimum very quickly. However, it can easily get trapped in a sub-space containing only solutions of poor quality. To diversify the search and force it to visit solutions with different characteristics, one basic idea was to increase the number of forbidden components when performing local modifications to a solution. So, the discussion quickly turned around the optimum taboo list size, since a short list allows a thorough examination of the neighbourhood of a good solution while a long list facilitates the escape from a local optimum to explore new regions of the search space. In fact, the reactive TS proposed by Battiti and Tecchiolli (1994) was designed to automatically adapt the taboo list size and avoid the fastidious task of explicitly managing the taboo list.

The main difficulty with TS is thus to efficiently incorporate diversification and intensification mechanisms. The use of a memory that stores good solutions visited during the search and the design of a procedure to create provisional solutions from it is a way to achieve this goal. Indeed, solutions contained in memory during the initial search phase present different characteristics, thus leading to a diversified search. Later, solutions contained in memory are mostly representative of one or a few good regions of the solution space. The result is that the search gradually shifts from diversification to intensification (Rochat and Taillard, 1995).

4. Conclusion and perspective

Our goal was to demonstrate that metaheuristics with memory have evolved toward a unified problem-solving approach that encompasses each one of them. Our point of view is that the name AMP is more general and better characterizes this unified approach than the name of any particular metaheuristic. The simplicity of the general AMP scheme could lead to think that one has reached the ultimate goal of metaheuristics: A unique technique that groups the essence of the good ideas contained in every metaheuristic. On the contrary, we think on the one hand that the reflections brought by this article should stimulate the creation of new metaheuristics based on completely different principles. On the other hand, designing an adaptive memory programme requires an intelligence that cannot be embedded in a metaheuristic.

This intelligence should be brought by the designer that has the knowledge of the problem to solve. First, we can think to the design of the procedure that constructs a provisional solution from solutions generated in the past. Undoubtedly, this construction requires knowledge about the problem. However, the design of the improving procedure could also benefit from future improvements. For example, it is reported in Taillard (1996) that a simple local search procedure, using an adequate neighbourhood of reasonable size, could produce solutions that were often of higher quality than previously designed taboo searches, GA or variable neighbourhood searches (Hansen and Mladenovic, 1999) for a class of clustering problems.

Finally, let us point out a few advantages of AMP. First, the technique is very well adapted for a parallel implementation on distributed computers (Badeau et al., 1997). Indeed, the most computationally expensive part of the algorithm is often the local search procedure. Therefore, it is

possible to place a local search process on many processors, each of them working on different provisional solutions. Second, AMP is particularly well adapted for solving dynamic problems since its working principle allows the method to adapt to slightly modified data (Gendreau et al., 1996, 1999). So, it is able to benefit from past computations while other methods like SA or variable neighbourhood searches "forget" the way they succeeded in finding – sometimes very efficiently – the unique solution they can exhibit. Finally, an adaptive memory programme is often able to exhibit a number of different high quality solutions. This enables the user to choose the practical solution that solves best his problem. Indeed, it is often not possible to include all features of a real application into an optimization model and the possibility of choosing a solution among a number of alternatives is of great importance in practice. For all these reasons, we think that the philosophy of AMP will be more and more adopted in the future, especially for real applications.

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