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# Convergence Performance Comparison of Quantum-inspired Multi-Objective Evolutionary Algorithms

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*Abstract*— **In recent research, we proposed a general framework of quantum-inspired multi-objective evolutionary algorithms(QMOEA) and gave one of its sufficient convergence conditions to Pareto optimal set. In this paper, two Q-gate operators, $H_\epsilon$ gate and $R\&N_\epsilon$ gate, are experimentally validated as two Q-gate paradigms meeting to the convergence condition. The former is a modified rotation gate, and the latter is a combination of rotation gate and NOT gate with the specified probability. To investigate their effectiveness and applicability, several experiments on the multi-objective 0/1 knapsack problems are carried out. Compared to two typical evolutionary algorithms and the QMOEA only with rotation gate, the QMOEA with $H_\epsilon$ gate and $R\&N_\epsilon$ gate have more powerful convergence ability in high complex instances. Moreover, the QMOEA with $R\&N_\epsilon$ gate has the best convergence in almost all of experiment problems. Furthermore, the appropriate $\varepsilon$ value regions for two Q-gates are verified.**

**Index Terms-**Multi-objective evolutionary algorithms, multi-objective 0/1 knapsack problems, quantum computing, convergence performance.

## I. Introduction

For the capability of searching simultaneously whole of solution spaces of multi-objective optimization problem (MOP) using a population of feasible solutions based on stochastic mechanisms, evolutionary algorithms have more advantage in dealing with discontinuous and concave Pareto fronts than traditional mathematical programming techniques. A large number of multi-objective evolutionary algorithms (MOEA) that employ some innovative mechanisms have been proposed during the last two decades, such as MOGA [1], NPGA [2], NSGA2 [3] and SPEA2 [4] etc. The last two have become very popular evolutionary algorithms for multi-objective optimization problems (MOP) in the last few years. Some important theoretical work related to MOEA has been done. Rudolph has investigated convergence properties of some MOEAs under partially ordered finite set theory [5], [6].

During the last decade, the quantum computational theory was attracting serious attention since their remarkable superiority of computational mechanical, which were demonstrated in Shor's quantum factoring algorithm [7] and Grover's database search algorithm [8]. Integrating the quantum computing mechanisms and classical evolutionary algorithms, some quantum-inspired evolutionary algorithms (QEA) were proposed in [9], [10], [11], [12] and [13], which are characterized by some quantum mechanics such as uncertainty, superposition, interference etc. Recently some quantum-inspired multi-objective evolutionary algorithms (QMOEA) combining MOEA with QEA were proposed to solve the multi-objective optimization problem (MOP) [14], [15]. In [16] we discussed a general framework of QMOEA and presented one of its convergence conditions.

In this paper, we test the convergence performance of QMOEAs with several different Q-gate strategies on the multi-objective 0/1 knapsack benchmark problems. Two Q-gate strategies, $H_\epsilon$ gate and $R\&N_\epsilon$ gate, which satisfy the convergence condition are experimented and discussed. As the referenced algorithms, the popular MOEAs, NSGA2 and SPEA2 , and the QMOEA with classic rotation gate are tested and evaluated simultaneously.

## II. The General Framework of QMOEA and its Convergence Condition

### A. The General Framework of Quantum-inspired Multi-objective Evolutionary Algorithms

Integrating the basic principle of quantum-inspired computing (QC) and general schemes of MOEA, we proposed a general framework of quantum-inspired multi-objective evolutionary algorithms in [16] as follows:

**Procedure of the QMOEAs' Basic Framework**

begin

$t \leftarrow 0$

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```
i)     initialize the Q-population Q(t)
ii)    the archives set A(t) = φ
iii)   while (not termination condition) do
        begin
          t ← t + 1
iv)       make P(t) by observing Q(t − 1)
v)        evaluate the solutions set P(t)
vi)       rebuild A(t) by selecting all of
           nodominated solutions from A(t − 1) ∪ P(t)
vii)      extract objectives solutions set O(t)
           from A(t) ∪ P(t)
viii)     make Q(t) by updating Q(t − 1) according
           to O(t) on one Q-gate strategy
        end
      end
```

In above procedure, those basic principle of quantum-inspired evolutionary computation are similar to those QEAs [12], [13] etc. Here we do not give the details. In MOEA some techniques are so successful that they have become general schemes, such as nondominated ranking, solutions diversity maintaining and elitism solutions archiving [17], and they are used in this framework. Especially, the elitists archives set $A(t)$ is crucial to its convergence property to Pareto front [5].

### *B. One of the Sufficient Convergence Conditions to QMOEAs*

Now let us see one of convergence conditions for this framework. Firstly, let $S$ be the feasible solution set for a MOP. $f$ is the function mapping the variable set $S$ to its image set and the comparing relation "$\preceq$" denotes "weak Pareto dominace" relation among those elements in the image set $f(S)$. According to those concepts and definitions about partially ordered set in [5], [6], we can look upon the image space, $(f(S), \preceq)$, as a partially ordered set. Here the set $M_f(S, \preceq)$ denotes the set of minimal elements of the image space $f(S)$, which equals to Pareto optimal set of the MOP. By the construction of the basic framework, $A(t)$ is the archives solutions set. We define the concept *convergence to the Pareto optimal set with probability 1* as follows [6]:

**Definition 2.1** *Let $F^* = M_f(S, \preceq)$ and $A(t)$ be the archives solutions set of QMOEA. The QMOEA is said to* converge with probability 1 *to the Pareto optimal set if*

$$\delta_{F^*}(f(A(t))) \to 0 \text{ with probability } 1 \text{ as } t \to \infty.$$

Here the measure function $\delta_A(B) = |A| - |A \cap B|$ means the number of elements that are in the set $A$ but not in the set $B$. The archives set $A(t)$ is defined by $M_f(A(t-1) \cup P(t), \preceq)$. Thus we can get one of the sufficient convergence conditions as follows:

**Theorem 2.2 ( Sufficient Convergence Condition )** *Let $S$ be a feasible solution set of MOP. One of the sufficient conditions by whose this QMOEA* converges with probability 1 to its Pareto optimal set *is that there exists a real number $\epsilon_0$, $0 < \epsilon_0 < 1$, which satisfies $Pro(s \in P(t)) \geq \epsilon_0$ for all $s \in S$, $t > 0$ and $Pro(s \in P(t))$ is* independent *from each other for different $t$.*

Here $Pro(s \in P(t))$ denotes the probability that the population $P(t)$ contains the solution $s$. The proofs for this theorem can be found in [18].

## III. Three Q-gate Strategies and their Convergence Properties

Based on the QMOEA's framework, we can use variant Q-gate strategies in viii) to construct a variety of algorithms. Now we discuss three Q-gate strategies and their convergence properties according to above theorem 1.

### *A. R Gate*

In QEA and QMOEA all of elements in $Q(t)$ are some Q-bits strings, and each Q-bit is built by a pair of number $(\alpha, \beta)$, which satisfy the normalization condition $|\alpha|^2 + |\beta|^2 = 1$. The rotation gate operator, here we call it $R$ gate, is the popular Q-gate strategy [12], [13] and [15], by which the updated Q-bit with a new pair of number $(\alpha', \beta')$ should satisfy the normalization condition $|\alpha'|^2 + |\beta'|^2 = 1$. The $R$ gate acting on a single Q-bit is the basic Q-gate in QMOEA as follows:

$$R(\Delta\theta) = \begin{bmatrix} \cos(\Delta\theta) & \sin(-\Delta\theta) \\ \sin(\Delta\theta) & \cos(\Delta\theta) \end{bmatrix}$$

where $\Delta\theta$ is a rotation angle toward either 0 or 1 state depending on its objective sign. As the $R$ gate is being applying to one Q-individual, a binary individual has been selected from the set $O(t)$ as the objective signs string. The objective sign to each bit of a Q-individual is defined as the corresponding bit of the objective solution, respectively. After the $R$ gate $R(\Delta\theta)$ acting on a Q-bit $(\alpha, \beta)$, the updated Q-bit $(\alpha', \beta')$ satisfy $(\alpha', \beta')' = R(\Delta\theta)(\alpha, \beta)'$.

Here $\Delta\theta$ should be designed in compliance with the application problem and each Q-bit possibly matches with different angles. Normally, a Q-bit acted by $R$ gate would increase the probability that observing result equals to its objective sign bit. Several rotation angle strategies can be referred in [10], [12] and [13].

Now let us discuss its convergence property of the QMOEA with $R$ gate. Because the $R$ gate has not any special technical guarantee of the convergence condition in *Theorem 2.2*, it may case that the QMOEA only with $R$ gate can not converge to its Pareto optimal set. However, we can show an example that can not converge to the Pareto optimal set in a local optimum situation as follows.

Let the procedure of this algorithm be defined as the basic framework in II-A, but the $R$ gate is used in the viii) step. The rotation angle $\Delta\theta$ parameter is similar with the description of *Table 1* in [12]. According to this $\Delta\theta$ parameter, it is clear that the Q-bit will hold its state if the observed bit of the Q-bit equals to its objective sign.

Let us assume that all solutions are presented by 4-bit binary digit string in one MOP. The feasible space $S = \{0, 1\}^4$, the Pareto optimal set $PS = \{1111, 1110\}$ and

"1111" dominate all elements in $S-PS$. In order to simplify the issue, we assume that $Q(t)$ and $P(t)$ are only contains one individual. Then we give a local optimum situation to show that the algorithm can not jump out from this situation and can not converge to the Pareto optimal set as follows:

In iteration $t_0$, if it is satisfied that

$$Q(t_0) = \left\{ q^{t_0} = \left( \begin{array}{c|c|c|c} \alpha_1^{t_0} & \alpha_2^{t_0} & 1 & \alpha_4^{t_0} \\ \beta_1^{t_0} & \beta_2^{t_0} & 0 & \beta_4^{t_0} \end{array} \right) \right\}$$

and $A(t_0) \cap$ "**1*" $= \phi$, then $A(t) \cap PS = \phi$ for any $t > t_0$.

By the observing operator, it is clear that $P(t_0+1) \cap$ "**1*" $= \phi$ because for the state of $Q(t_0)$. From the definition $A(t+1) = M_f(A(t) \cup P(t+1))$, we can get that $A(t_0+1) \cap$ "**1*" $= \phi$. Hence $A(t_0+1) \cap PS = \phi$ for $PS = \{1111, 1110\}$. As the objective solution set $O(t_0+1) = \{o(t_0+1) \in A(t_0+1)\}$, it is sure that $O(t_0+1) \cap$ "**1*" $= \phi$, too. By rotation gate updating operator with $\Delta\theta$ parameter as above text, we can conclude that the 3rd Q-bit of $Q(t_0+1)$ will be held on $(1, 0)$. Thus in iteration $t_0+1$, it is satisfied that

$$Q(t_0+1) = \left\{ q^{t_0+1} = \left( \begin{array}{c|c|c|c} \alpha_1^{t_0+1} & \alpha_2^{t_0+1} & 1 & \alpha_4^{t_0+1} \\ \beta_1^{t_0+1} & \beta_2^{t_0+1} & 0 & \beta_4^{t_0+1} \end{array} \right) \right\}$$

and $A(t_0+1) \cap$ "**1*" $= \phi$. We can repeat the deduction as above continuously. So the algorithm will be immersed in this situation and can not converge to the Pareto optimal set forever.

So we say that the QMOEA only with $R$ gate have not a guarantee of converging to the Pareto optimal set.

### B. $H_\epsilon$ Gate

The $R$ gate induces the convergence of each Q-bit to either 0 or 1. However, sometimes a Q-bit converged to either 0 or 1 cannot escape the state by itself. To prevent the premature convergence of Q-bit due to $R$ gate, a modified Q-gate is defined as $H_\epsilon$ *gate*, which is extended from the $R$ *gate* and was proposed by Han and Kim in [13]. If acted by $H_\epsilon$ gate, a Q-bit $(\alpha, \beta)$ would be updated as

$$(\alpha', \beta') = H_\epsilon(\alpha, \beta, \Delta\theta),$$

where for $(\alpha'', \beta'')' = R(\Delta\theta)(\alpha, \beta)'$

1) if $|\alpha''|^2 \le \epsilon$ and $|\beta''|^2 \ge 1-\epsilon$ then
$$(\alpha', \beta')' = (\sqrt{\epsilon}, \sqrt{1-\epsilon})';$$
2) if $|\alpha''|^2 \ge 1-\epsilon$ and $|\beta''|^2 \le \epsilon$ then
$$(\alpha', \beta')' = (\sqrt{1-\epsilon}, \sqrt{\epsilon})';$$
3) otherwise $(\alpha', \beta')' = (\alpha'', \beta'')'$.
where $0 < \epsilon \ll 1$ and $R(\Delta\theta)$ is a $R$ gate as describing in III-A .

The procedure of QMOEA with $H_\epsilon$ gate is similar with that basic framework in II-A but the $H_\epsilon$ gate is adopted in step viii). This algorithm can converge with probability 1 to its Pareto optimal set due to the definition of $H_\epsilon$ gate as above. We explain it simply as follows.

We assume $Q(t) = \{q_i^t | i = 1, 2, \ldots n\}$ where

$$q_i^t = \left( \begin{array}{c|c|c|c} \alpha_{i1}^t & \alpha_{i2}^t & \cdots & \alpha_{im}^t \\ \beta_{i1}^t & \beta_{i2}^t & \cdots & \beta_{im}^t \end{array} \right).$$

For the definition of $H_\epsilon$ gate as above, it is clear that $\epsilon \le |\alpha_{ij}^t|^2 \le 1-\epsilon$ and $\epsilon \le |\beta_{ij}^t|^2 \le 1-\epsilon$ for all $i = 1, 2, \ldots n$, $j = 1, 2, \ldots m$ and $t > 0$. By using the observing operator, the probability that any one of solutions $s$ exist in binary population $P(t)$ can be estimated as follows:

$$\begin{aligned} Pro(s \in P(t)) =& 1 - Pro(s \ not \in P(t)) \\ =& 1 - \prod_{i=1}^{n} (1 - Pro(Observing\ q_i^t = s)) \\ \ge& 1 - (1-\epsilon^m)^n \end{aligned}$$

Let $\epsilon_0 = 1-(1-\epsilon^m)^n$. Thus, we can say the algorithm can converge to its Pareto optimal set according to the theorem 2.2.

### C. $R\&N_\epsilon$ Gate

The $NOT$ gate is one of classic quantum gates, which function is to exchange Q-bit's parameters. The $N_\epsilon$ gate is a $NOT$ gate with the probability $\epsilon$. Its transformation matrix can be defined as follows:

$$N_\epsilon : \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \text{with probability} \quad \epsilon;$$
$$N_\epsilon : \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \text{with probability} \quad 1-\epsilon;$$

where $0 < \epsilon \ll 1$. $R\&N_\epsilon$ gate is composed of the $R$ gate and the $N_\epsilon$ gate. If acted by $R\&N_\epsilon$ gate, a Q-bit $(\alpha, \beta)$ would be updated as

$$(\alpha', \beta') = R\&N_\epsilon(\alpha, \beta, \Delta\theta),$$

where for $(\alpha'', \beta'')' = R(\Delta\theta)(\alpha, \beta)'$ and $\varepsilon'' = rand()$

1) if $\varepsilon'' \le \epsilon$ then $(\alpha', \beta')' = (\beta'', \alpha'')'$;
2) if $\varepsilon'' > \epsilon$ then $(\alpha', \beta')' = (\alpha'', \beta'')'$;

where $rand()$ denotes the random function whose value is in $[0, 1]$, and $R(\Delta\theta)$ is a $R$ gate as describing in III-A .

The QMOEA with $R\&N_\epsilon$ gate is too similar with that basic framework in II-A but the $R\&N_\epsilon$ gate is adopted in step viii). This algorithm can converge to its Pareto front.

Because of the definition of $R\&N_\epsilon$ gate as above, for any Q-bit $q_{ij}^t = (\alpha', \beta')$ we can conclude that

$$\begin{aligned} Pro(Observing\ q_{ij}^t = 0) =& (1-\epsilon)|\alpha''|^2 + \epsilon|\beta''|^2 \\ =& (1-2\epsilon)|\alpha''|^2 + \epsilon \\ \ge& \epsilon. \end{aligned}$$

For the same reasoning, we can get $Pro(Observing\ q_{ij}^t = 1) \ge \epsilon$ for all $i = 1, 2, \ldots n$, $j = 1, 2, \ldots m$ and $t > 0$. So the probability that any one of solutions $s$ exist in binary population $P(t)$ can be estimated as follows:

$$\begin{aligned} Pro(s \in P(t)) =& 1 - Pro(s \ not \in P(t)) \\ =& 1 - \prod_{i=1}^{n} (1 - Pro(Observing\ q_i^t = s)) \\ \ge& 1 - (1-\epsilon^m)^n. \end{aligned}$$

Let $\epsilon_0 = 1-(1-\epsilon^m)^n$. Thus, we can say the algorithm can converge to its Pareto optimal set according to the theorem 2.2, too.

## IV. Benchmark Problems and Experimental Methodology

In the following, the benchmark experiment study is described that has been carried out using those QMOEAs with three above Q-gate strategies for solving the multi-objective 0/1 knapsack problems. In this paper, the comparison focuses on the convergence effectiveness in finding multiple Pareto optimal solutions. As additional references with classic MOEAs, the NSGA2 and SPEA2 have carried out here, too.

### A. Benchmark Problems

*1) The Multi-objective 0/1 Knapsack Problem:* The Knapsack Problem is easy to describe and understand, it represent a certain class of real world problems. However, it is difficult to solve (NP hard). So the multi-objective 0/1 knapsack problem has been as a good benchmark problem in evaluating some multi-objective optimization algorithms [3], [4], and [15] etc.

A single objective 0/1 knapsack problem consists of a set of items, and a pair of weight and profit for each item and an upper bound for the capacity of the knapsack are given. The task is to find a subset of items which maximizes the total of the profits in the subset, yet the total weight of all selected items into the knapsack does not exceed the given capacity. Here we extend the single objective 0/1 knapsack problem to the multi-objective case, which contains of an arbitrary number of knapsacks. Formally, the multi-objective knapsack problem is defined as the following:

Given a set of $m$ items and a set of $n$ knapsacks, the problem is to find a vector $\mathbf{x} = (x_1, x_2, \ldots, x_m) \in \{0,1\}^m$, such that

$$\sum_{j=1}^{m} w_{i,j} \cdot x_j \le c_i, \quad \text{for all} \quad 1 \le i \le n$$

and for which $f(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots\cdots, f_n(\mathbf{x}))$ is maximum, where

$$f_i(\mathbf{x}) = \sum_{j=1}^{m} p_{i,j} \cdot x_j, \quad \text{for all} \quad 1 \le i \le n,$$

and $p_{i,j}$ is profit of item $j$ according to knapsack $i$, $w_{i,j}$ is weight of item $j$ according to knapsack $i$, $c_i$ is capacity of knapsack $i$.

*2) Test Data:* In experiments we used nine different test problems where both the number of knapsacks and the number of items were varied. Two, three and four objectives were taken under consideration, in combination with 250,500 and 750 items.

In order to compare with those classic MOEAs, we used those test data sets in [19]. Those profits $p_{i,j}$ and weights $w_{i,j}$ were random integers in the interval $[10, 100]$. The knapsack capacities were set to half the total weight regarding the corresponding knapsack.

### B. Implementation and Performance Measures

*1) $\Delta\theta$ Parameter:* The influence of $\Delta\theta$ parameter is significant on convergence effectiveness to the Pareto front for $R$ gate, $H_\epsilon$ gate and $R\&N_\epsilon$ gate. It should be designed in compliance with the application problem. On single objective knapsack problem optimization, Han and Kim gave a $\Delta\theta$ parameter setting in [12]. Here we set a new setting for multi-objective 0/1 knapsack problem optimization problem as follows.

TABLE I

The $|\Delta\theta|$ Parameter Lookup Table, where $f(\cdot)$ is the profit, and $b_i$ and $x_i$ are the $i$th bit of the binary objective solution $\mathbf{b}$ and the binary observing solution $\mathbf{x}$ associated with one quantum individual $q(t) \in Q(t)$, respectively.

| $x_i$ | $b_i$ | $f(\mathbf{b}) \preceq f(\mathbf{x})$ | $\|\Delta\theta\|$ |
|---|---|---|---|
| 0 | 1 | $true$ | $0.04\pi$ |
| 0 | 1 | $false$ | $0.02\pi$ |
| 1 | 0 | $true$ | $0.02\pi$ |
| 1 | 0 | $false$ | $0.01\pi$ |
| 1 | 1 | $true$ | $0.02\pi$ |
| 1 | 1 | $false$ | $0.01\pi$ |
| 0 | 0 | $true$ | $0.01\pi$ |
| 0 | 0 | $false$ | 0 |

The angle $|\Delta\theta|$ parameter used for the rotation gate in three Q-gate strategy are shown in Table I. However, the value of $\Delta\theta$ is determined by $|\Delta\theta|$, $b_i$ and the state of the Q-bit as follows:

1) If $b_i$ is 0 and the Q-bit is located in the first or the third quadrant, or $b_i$ is 1 and the Q-bit is located in the second or the fourth quadrant; then the value of $\Delta\theta$ is set to a negative value $\Delta\theta = -|\Delta\theta|$;
2) if $b_i$ is 0 and the Q-bit is located in the second or the fourth quadrant, or $b_i$ is 1 and the Q-bit is located in the first or the third quadrant; then the value of $\Delta\theta$ is set to a positive value $\Delta\theta = |\Delta\theta|$.

In fact, we have experimented several different $\Delta\theta$ settings schemas in order to choose some more reasonable ones. However, the above setting may be one of the best choices as far as the experimental results. Thus we adopted it for the rotation angle setting of all three Q-gate strategies in the following experiments.

*2) Objectives Assignment Schema:* When the Q-individuals are updated by Q-gates in the viii) step of the basic framework, each Q-individual should be assigned one objective solution in $O(t)$ as the referenced signs string. One type of sorted individuals grouping schema according to objectives is described and tested for the multi-objective 0/1 knapsack problem in [15]. In our experiments, five type of grouping classification schemas for objectives assignment were tested as follows:

*OAS1*: Both all of individuals in $P(t)$ and objectives in $O(t)$ are descending sorted based on nondominated rank and crowded distance partial order. Then the $Q(t)$ is divided into several groups by the order associated with $P(t)$. Each group in $Q(t)$ is assigned one objective solution from $O(t)$

according to their orders. The each assigned objective solution is the referencing signs string to all Q-individuals in the corresponding group. This method is described in [15].

*OAS2*: Firstly, we compte the Hamming distances in decision space between each individual in $P(t)$ and each objective solution in $O(t)$. Then all individuals in $P(t)$ are divided into several clusters round objective solution in $O(t)$ according to the Hamming distances. Every clusters have almost same number of members. Thus each objective solution in $O(t)$ is assigned the referencing signs string to those Q-individuals in $Q(t)$ associated with the relevant cluster of $P(t)$.

*OAS3*: This method is similar with the *OAS2* but the Hamming distances in decision space is replaced by the Euler distances in objective space.

*OAS4*: This method is to select one objective solution from $O(t)$ as the sign string for each Q-individual in $Q(t)$ by the binary tournament selection according to nondominated rank and crowding distance partial order.

*OAS5*: Certainly, Randomly selecting one objective solution from $O(t)$ as the sign string for each Q-individual in $Q(t)$ is one of feasible methods, too.

Because of *OAS2* with better experimental effect, we applied it as the objectives assignment schema in the following experiments.

*3) Repairing Method:* As the constraints of knapsack capacities, all infeasible binary solutions in $P(t)$ should be repaired before being evaluated. A greedy repair method was used in [19]. Here we used a improved greedy repair method to produce the best repaired outcomes from original solutions. This repairing algorithm try to repair all infeasible solution such that they fulfill all the capacity constraints and reserve the overall profits as maximum as possible.

In particular, a infeasible binary solution $\mathbf{x}$ in $P(t)$ can be look upon as a string $s$ of length $m$. Firstly, all items by their maximum profit/weight ratio per item are sorted, for item $j$ the maximum profit/weight ratio $r_j$ is defined as $r_j = \max_{i=1}^{n}\{p_{i,j}/w_{i,j}\}$. Then, those items from string $s$ are removed step by step according to the $r_j$ ascending order until all capacity constraints are fulfilled. Finally, those items whose maximum profit/weight ratio are smaller than the last removed item try to be added into $s$ step by step according to the $r_j$ descending order as long as all capacity constraints are fulfilled for the repaired $s$.

In each step above, the lowest profit per weight item is removed or the highest profit per weight item is added. This mechanism intends to fulfill the capacity constraints while the overall profit be reserved as more as possible.

*4) Performance Measures Methodology:* In this comparison the focus is on the convergence effectiveness to the Pareto optimal set, otherwise the distribution of the nondominated solutions along the trade-off surface is taken into account. So two scaling-independent metrics were used to evaluate the trade-off fronts produced by the various algorithms: the size of the nondominated objective space $S$ and the diversity of the nondominated solutions set $D$.

Given a points set $\{x_1, x_2, \cdots, x_k\}$, this first measure metric $S(x_1, x_2, \cdots, x_k)$ gives the volume enclosed by the union of the polytopes $p_1, p_2, \cdots, p_k$ where each $p_i$ is formed by those hyperplanes arising out of $x_i$ along with the axes: for each axis in the objective space, there exists a hyperplane perpendicular to the axis and passing through the point [19].Those algorithms with larger $S$ value of the nondominated solutions set mean better convergence to the global Pareto optimal front.

The second measure was employed to evaluate the diversity of nondominated solutions set. The metric is given as follows:

$$D = \frac{\sum_{k=1}^{n}\left(f_k^{(max)} - f_k^{(min)}\right)}{\sqrt{\frac{1}{|N_0|}\sum_{i=1}^{|N_0|}\left(d_i - \overline{d}\right)^2}}$$

Here $N_0$ is a set of nondominated solutions, $d_i$ denotes the minimal distance between the $i^{th}$ solution and the nearest neighbor, $d$ is the mean value of all $d_i$. $f_k^{(max)}$ and $(f_k^{(max)})$ represent respectively the maximum and minimum fitness of the $k^{th}$ objective. Those nondominated solutions sets with larger $D$ value have higher diversity.

### C. Verification of $\varepsilon$ Values Selection

Here we verify the selection of $\varepsilon$ values for $R\&N_\epsilon$ gate and $H_\epsilon$ gate. The $\varepsilon$ value is significant to the convergence properties of QMOEAs with $H_\epsilon$ gate and $R\&N_\epsilon$ gate. Clearly, the appropriate $\varepsilon$ value will cause the better convergence performance in QMOEAs. Thus, we did experimental verification of $\varepsilon$ values for the reasonable value range. We chose the multi-objective 0/1 knapsack problem with 250 items and 2 knapsacks as the benchmark optimization problem. Figure 1 and 2 show the mean of $S$, which denotes size of nondominated objctive space in the end, in 30 runs for each $\varepsilon$ value. Other parameter values except $\varepsilon$ are same as Table II.

From experimental results in the Figure 1, $\varepsilon$ could be assigned in $[0.005, 0.05]$, where QMOEA with $H_\epsilon$ gate can converge better. Furthermore, $0.03$ seems to be one of the most appropriate ones. Similarly by the experimental results in the Figure 2, we may say that $[0.003, 0.02]$ would be the appropriate $\epsilon$ value region for QMOEA with $R\&N_\epsilon$ gate and $0.0075$ maybe one of the best choices.

## V. Experiments Results

### A. Parameters Setting

We chose two/three/four-knapsacks with 250/500/750 items as the benchmark problems setting. Three QMOEAs, respectively with $R$ gate, $H_\epsilon$ gate and $R\&N_\epsilon$ gate, were carried out. Here two outstanding MOEAs, SPEA2 and NSGA2, were chosen as the reference algorithms, where a pair-wise tournament selection, binary-coded GA with 1-point crossover and bitwise mutation were in force for them. Parameters used in this experiment are given in Table II, where $l$ is the length of binary string.

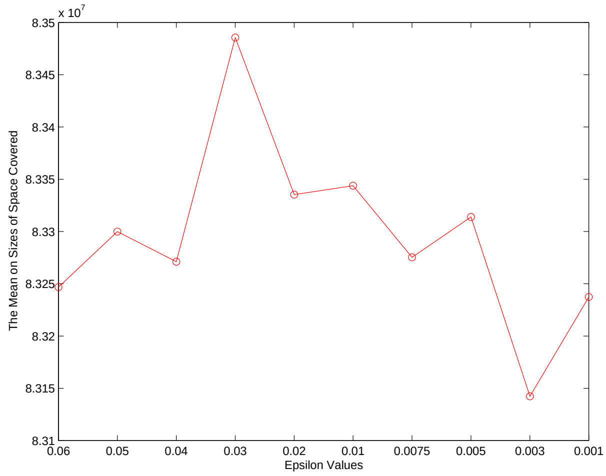


Fig. 1. The mean of convergence performance $S$ values in 30 runs for QMOEA with $H_\epsilon$ gate in different $\varepsilon$ values on MKP with 250 items and 2 knapsacks.

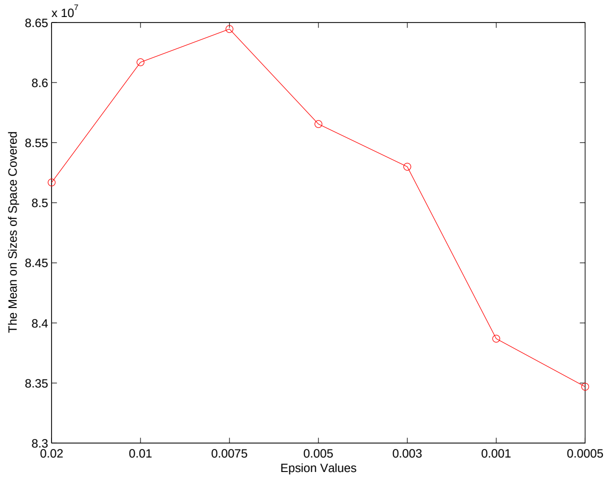


Fig. 2. The mean of convergence performance $S$ values in 30 runs for QMOEA with $R\&N_\epsilon$ gate in different $\varepsilon$ values on MKP with 250 items and 2 knapsacks.

For comparison of their convergence effectiveness, the population size and maximum archive set size were fixed at 100 for all algorithms. Each Q-individual in QMOEAs was observed only one time and the maximum iteration was fixed at 500 such that the numbers of objective function evaluation were same to all tested algorithms in one run. Here $\varepsilon$ values for $H_\epsilon$ gate and $R\&N_\epsilon$ gate were obtained from experimental evaluations. The experimental verification of $\varepsilon$ values will be presented in section IV-C. The crossover probability and mutation probability were gotten from parameters setting described in [3] and [4].

### B. Experiments Results

In this study, 30 independent runs were performed per algorithms and per problems. For each run, we measure their $S$ values and $D$ values over time, which leads to two samples of 30 values in each step of each experiment run. The plots on $S$ and $D$ show as the following fig 3 to fig 11. The graphs

TABLE II

PARAMETERS TABLE

| Parameter | Value |
|---|---|
| Population Size for $Q(t)$ | 100 |
| Population Size for $P(t)$ | 100 |
| Maximum Archive Size of $A(t)$ | 100 |
| Size for $O(t)$ | 10 |
| Observing Times | 1 |
| Maximum Iteration | 500 |
| $\varepsilon$ for $H_\epsilon$ gate | 0.03 |
| $\varepsilon$ for $R\&N_\epsilon$ gate | 0.0075 |
| Crossover Probability | 0.9 |
| Mutation Probability | $1/l$ |

show that the average $S$ and $D$ values of 30 runs for each algorithm over time and the distributions of the 30 $S$ values at the end of the runs.

In the experiments results, two excellent methods, NSGA2 and SPEA2 were considered in this comparison as additional points of reference. The results concerning the $S$ measure (size of the space covered) and and $D$ measure (diversity of the archive set) are respectively shown in the above and below part of all Figures. Here box plots (the middle part of all figures) are used to visualize the distribution of $S$ measure at the end of all 30 runs.

The simulation results prove that the QMOEA with $R\&N_\epsilon$ gate do better convergence than all other algorithms. These figures shows that its $S$ mean values over time outperform entirely others in all test problems. Especially, this algorithm can get very outstanding $S$ measure mean in the early running. Furthermore, the box plots show that its $S$ medians of 30 runs are superior to others at the end of 50000 objective function evaluations for all test problems. This indicates that the QMOEA with $R\&N_\epsilon$ gate has best convergence over time of 50000 objective function evaluations running, especially in the early.

In the whole running process, the $S$ measure of QMOEA with $H_\epsilon$ gate and $R$ gate are inferior to others. And the former is slight superior to the latter. However, all QMOEAs have better $S$ measure in the early of running, such as before 5000 objective function evaluations for MKP with 250 items and 2 knapsacks, before 10000 objective function evaluations for MKP with 500 items and 4 knapsacks, before 17000 objective function evaluations for MKP with 750 items and 4 knapsacks etc.

Considered the $D$ measure, the QMOEA with $R\&N_\epsilon$ gate has similar performance with NSGA2, and it is superior to SPEA2. But QMOEA with $H_\epsilon$ gate and $R$ gate are inferior to other algorithms.

From all experiments results, it is clear that QMOEA with $R\&N_\epsilon$ gate has the best convergence performance over running time among all algorithms, especially in the early running. Meanwhile, its diversity performance of nondominated solution set is similar to NSGA2 over running time. Moreover, all QMOEA can better convergence performance in early running time.

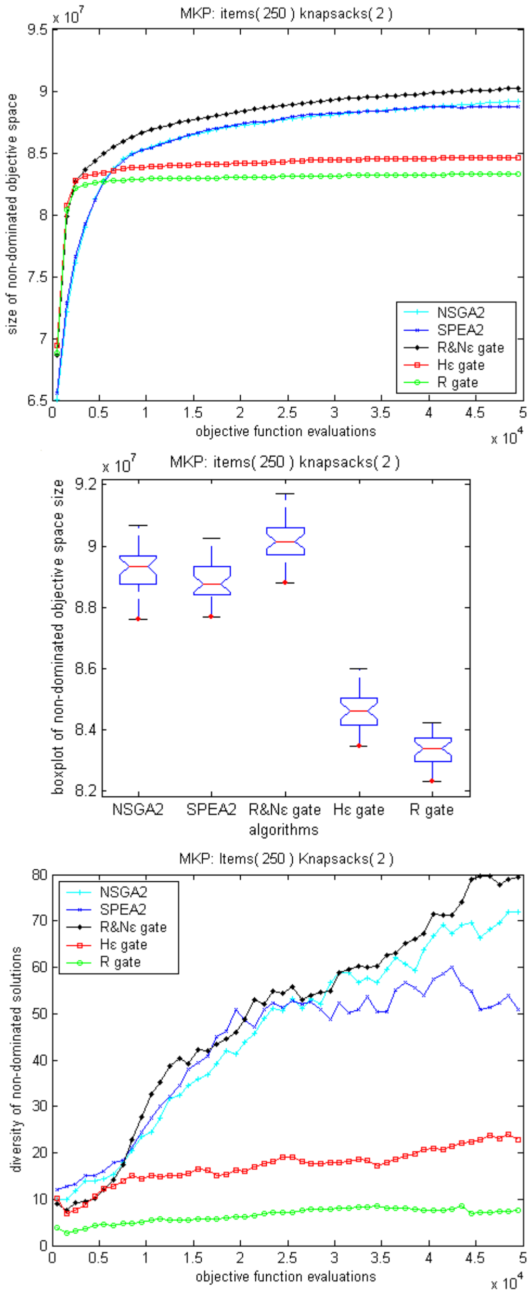


Fig. 3. Convergence and diversity performance for multi-objective 1/0 knapsack problems with 250 items and 2 knapsacks.

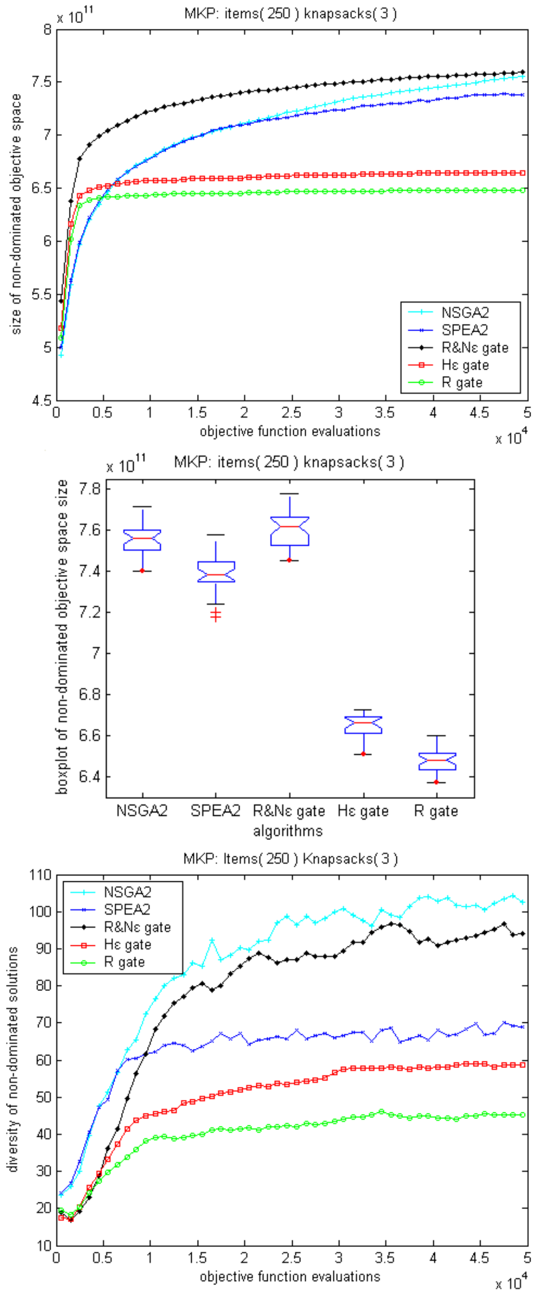


Fig. 4. Convergence and diversity performance for multi-objective 1/0 knapsack problems with 250 items and 3 knapsacks.

## VI. Conclusions

We have proposed a framework of quantum-inspired multi-objective evolutionary algorithms and given a sufficient convergence condition in recent work. In this paper, we discuss the convergence properties on three type of Q-gates: $R$ gate, $H_\epsilon$ gate and $R\&N_\epsilon$ gate. Those QMOEAs with two latter Q-gates can converge to the Pareto optimal set, but the former maybe be immersed in the local optimum situation instead of global optimum situation.

The multi-objective 0/1 knapsack problems optimization experiments were carried out as the benchmark problems for three type of Q-gates. Two classic MOEAs, SPEA2 and NSGA2, were as the referencing algorithms. The experimental results supported the following conclusions:

1) The QMOEA with $R\&N_\epsilon$ gate do best convergence among all algorithms over whole running time, especially in the early running. And this algorithm can get similar diversity of nondominated solutions set to NSGA2.
2) The QMOEA with $H_\epsilon$ gate and $R$ gate are inferior to others at the point of convergence performance. However the former is slight superior to the latter.
3) In the early running time, all QMOEAs are superior to

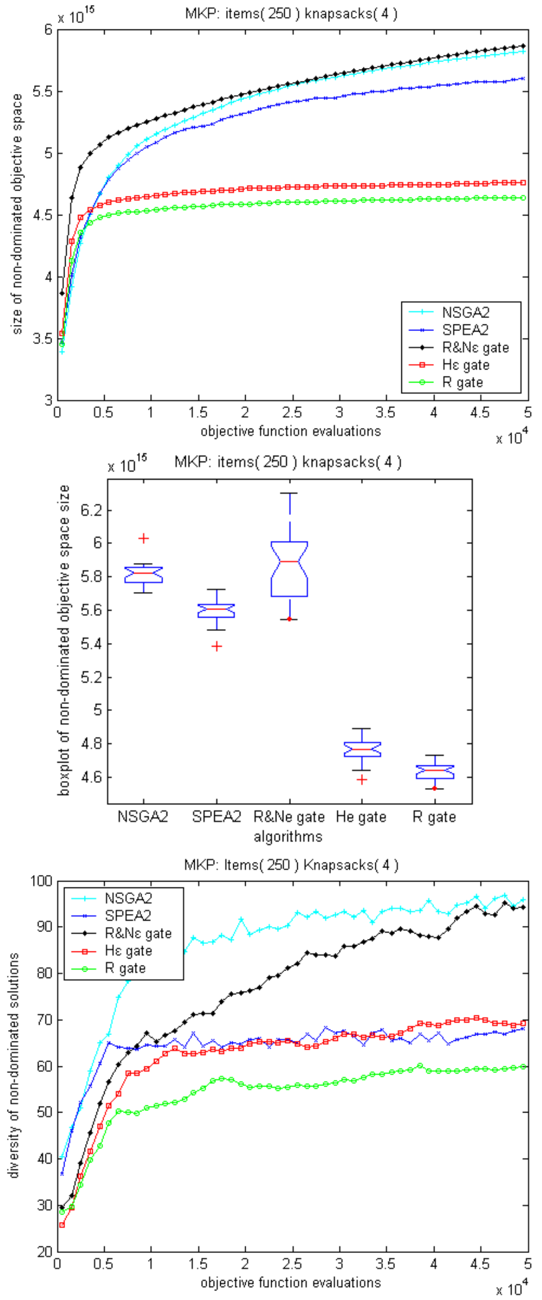


Fig. 5. Convergence and diversity performance for multi-objective 1/0 knapsack problems with 250 items and 4 knapsacks.

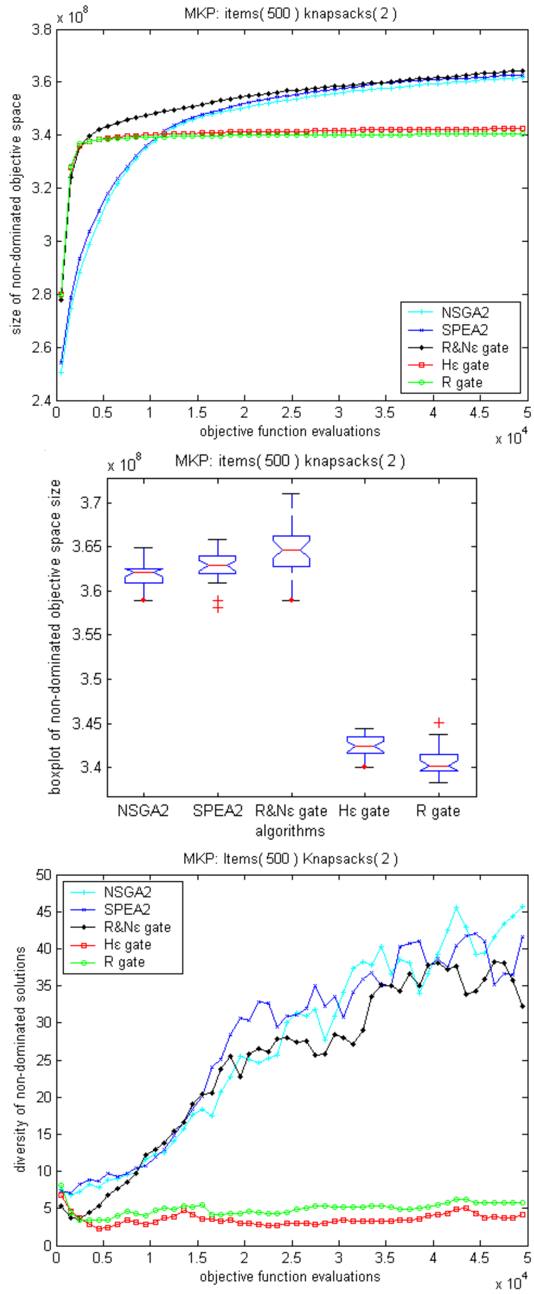


Fig. 6. Convergence and diversity performance for multi-objective 1/0 knapsack problems with 500 items and 2 knapsacks.

NSGA2 and SPEA2 for all test problems, especially for those more complex and high-dimension ones.

Furthermore, we verified the appropriate $\varepsilon$ value regions for two Q-gates, $H_\epsilon$ gate and $R\&N_\epsilon$ gate.

From these experiments results, we can deduce that the quantum-inspired mechanism could cause better convergence in the early running and the QMOEA with appropriate quantum gate mechanism, such as $R\&N_\epsilon$ gate, may be a better choice to some complex multi-objective combination optimization problems.

In the next step of research on QMOEAs, some further topics would be interested, such as convergence, robustness, efficiency and so on. Of course, their application in practical problems would be a part of the future work, for example bio-informatics, online scheduling, optimization with dynamic environment etc.

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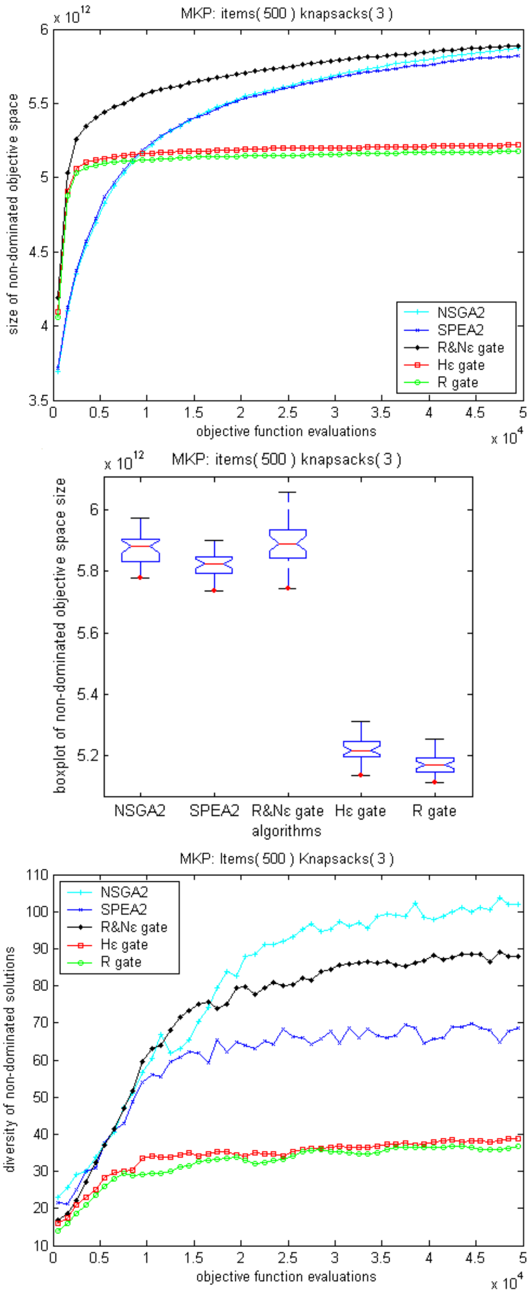


Fig. 7. Convergence and diversity performance for multi-objective 1/0 knapsack problems with 500 items and 3 knapsacks.

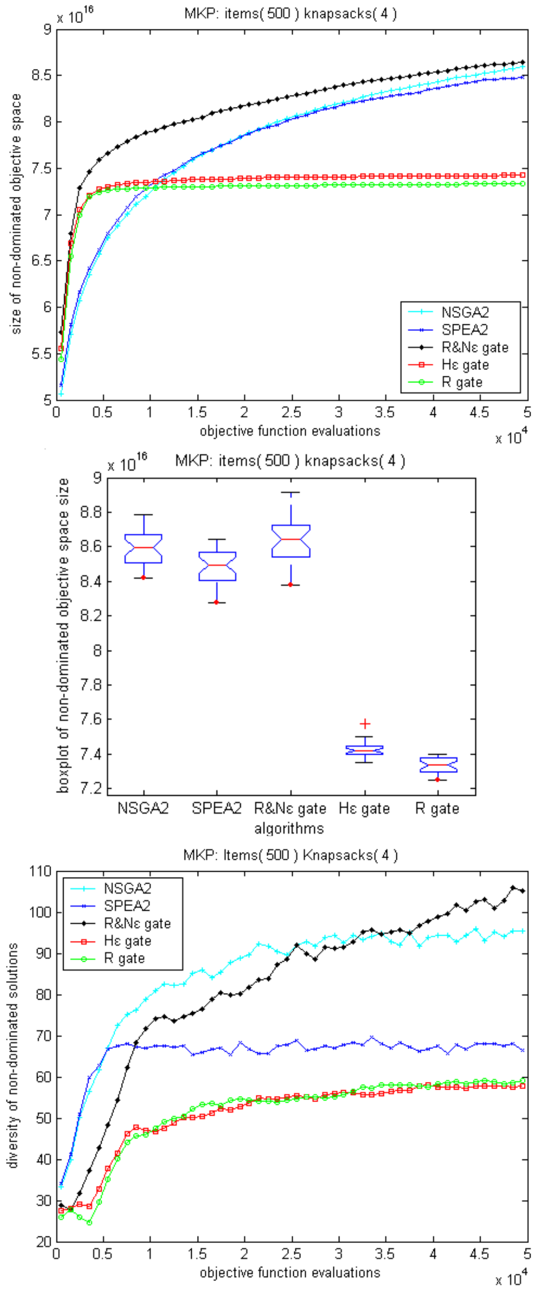


Fig. 8. Convergence and diversity performance for multi-objective 1/0 knapsack problems with 500 items and 4 knapsacks.

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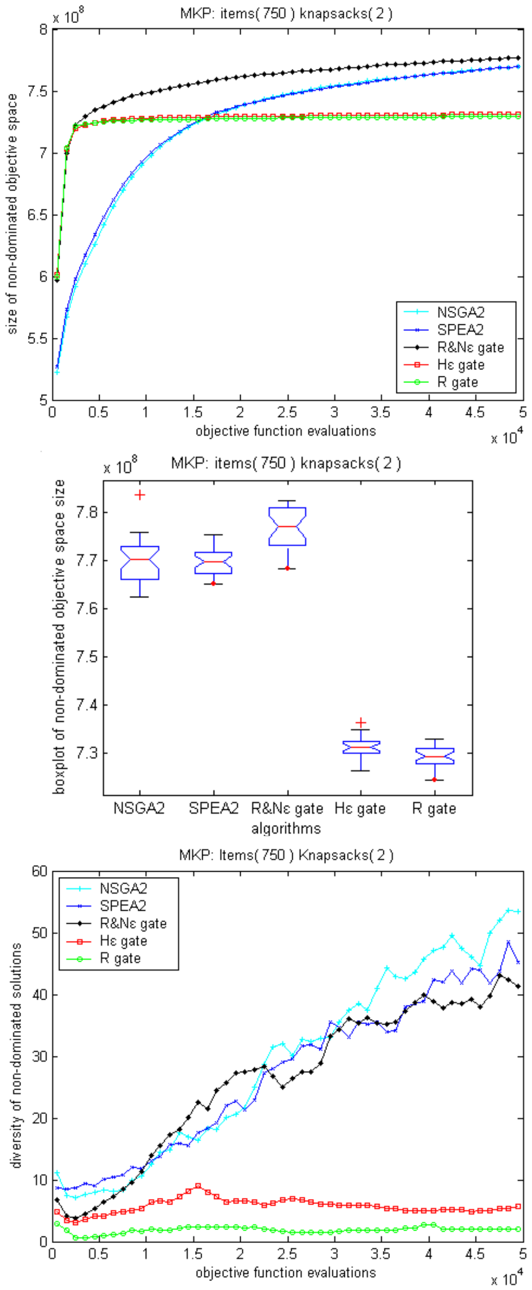


Fig. 9. Convergence and diversity performance for multi-objective 1/0 knapsack problems with 750 items and 2 knapsacks.

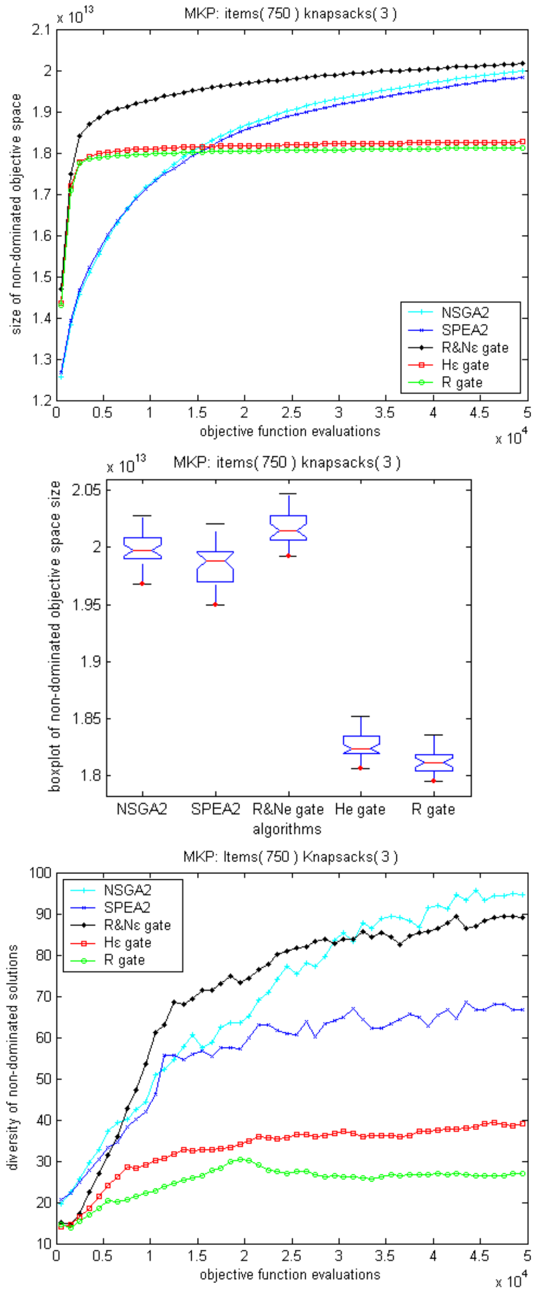


Fig. 10. Convergence and diversity performance for multi-objective 1/0 knapsack problems with 750 items and 3 knapsacks.

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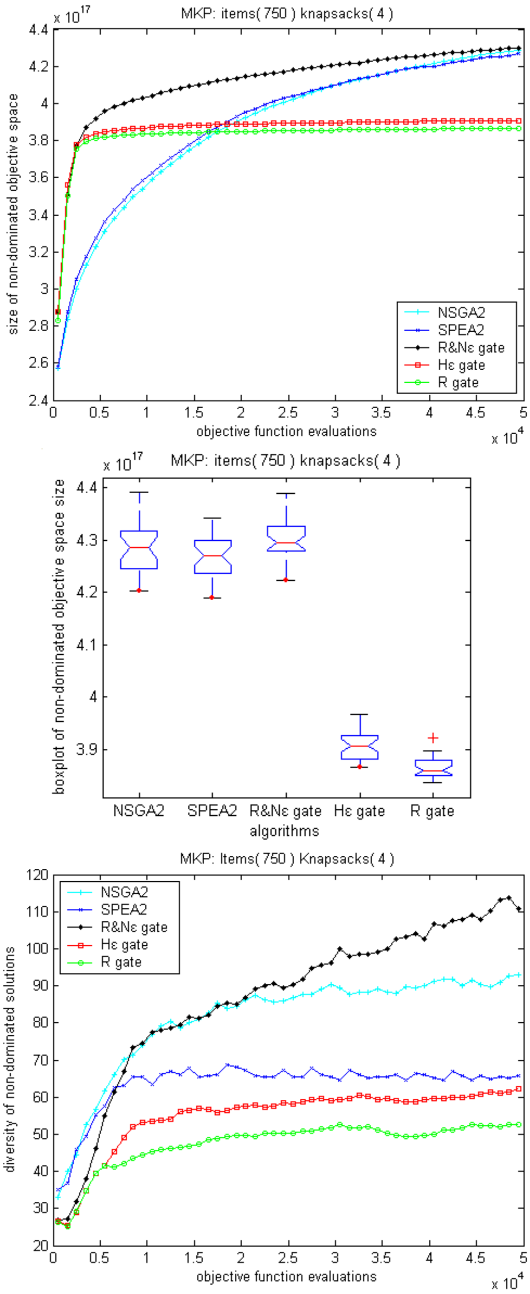


Fig. 11. Convergence and diversity performance for multi-objective 1/0 knapsack problems with 750 items and 4 knapsacks.


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